'(1,2,12,13)
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$n_0, m_1, m_4,$
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$n_0, m_1, m_4,$
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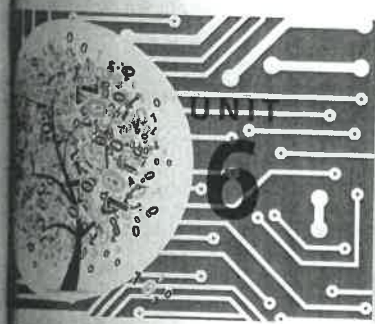


UNIT 6

Quine-McCluskey Method

Objectives

1. Find the prime implicants of a function by using the Quine-McCluskey method. Explain the reasons for the procedures used.
2. Define *prime implicant* and *essential prime implicant*.
3. Given the prime implicants, find the essential prime implicants and a minimum sum-of-products expression for a function, using a prime implicant chart and using Petrick's method.
4. Minimize an incompletely specified function, using the Quine-McCluskey method.
5. Find a minimum sum-of-products expression for a function, using the method of map-entered variables.

Study Guide

1. Review Section 5.1, *Minimum Forms of Switching Functions*.
2. Read the introduction to this unit and, then, study Section 6.1. *Determination of Prime Implicants*.

   (a) Using variables A, B, C, D, and E, give the algebraic equivalent of
   10110 + 10010 = 10–10
   10–10 + 10–11 = 10–1–

   (b) Why will the following pairs of terms not combine?
   01101 + 00111
   10–10 + 001–0

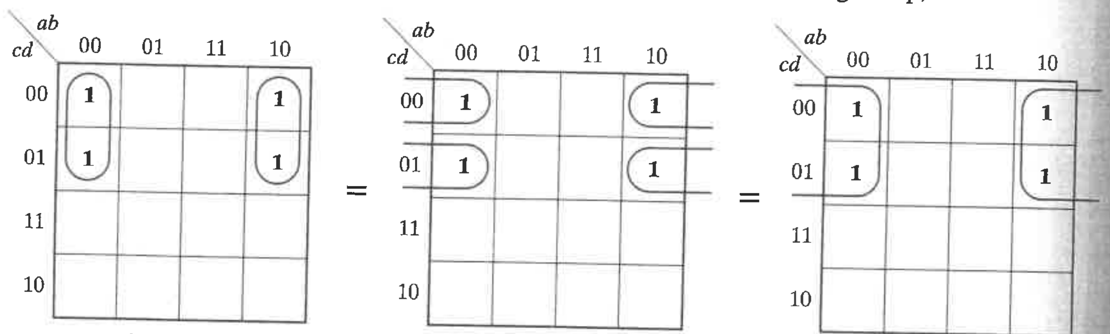
   (c) When using the Quine-McCluskey method for finding prime implicants, why is it necessary to compare terms only from adjacent groups?

   (d) How can you determine if two minterms from adjacent groups will combine by looking at their decimal representations?

   (e) When combining terms, why is it permissible to use a term which has already been checked off?

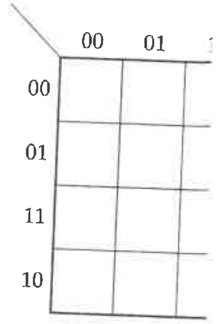
   (f) In forming Column II of Table 6-1, note that terms 10 and 14 were combined to form 10, 14 even though both 10 and 14 had already been checked off. If this had not been done, which term in Column II could not be eliminated (checked off)?

   (g) In forming Column III of Table 6-1, note that minterms 0, 1, 8, and 9 were combined in two different ways to form –00–. This is equivalent to looping the minterms in two different ways on the Karnaugh map, as shown.



(0, 1) + (8, 9) = (0, 8) + (1, 9) = (0, 1, 8, 9)

(h) Using a map, find
pare your answer



(i) The prime implica
be found using th
Column IV and ch

C
(4, 5, 6, 7)
(4, 5, 12, 13)
(4, 6, 12, 14)
(5, 7, 13, 15)
(6, 7, 14, 15)
(12, 13, 14, 15)

Check your answer usin

3. (a) List all seven produ

   Which of these imp

   Why is $a'c$ not an im

   (b) Define a prime impl

   (c) Why must every te
   prime implicant?

um Forms of Switching Functions.

his unit and, then, study Section 6.1. *Determination of*

C, D, and E, give the algebraic equivalent of
-10
-1–

g pairs of terms not combine?

e-McCluskey method for finding prime implicants,
compare terms only from adjacent groups?

e if two minterms from adjacent groups will combine
imal representations?

is, why is it permissible to use a term which has
off?

of Table 6-1, note that terms 10 and 14 were com-
en though both 10 and 14 had already been checked
done, which term in Column II could not be elimi-

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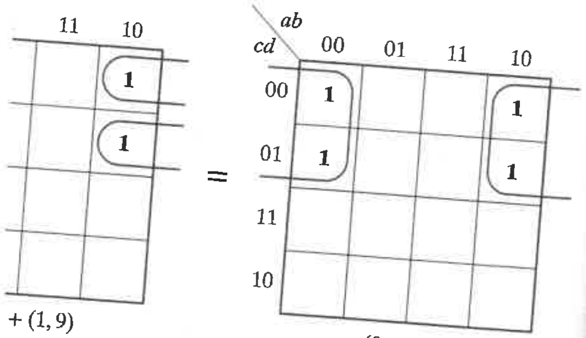
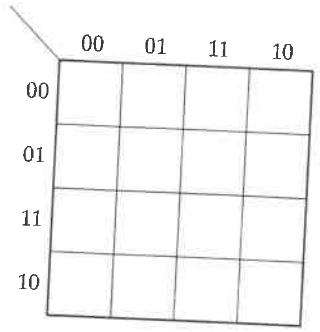
nt ways to form –00–. This is equivalent to looping
ferent ways on the Karnaugh map, as shown.



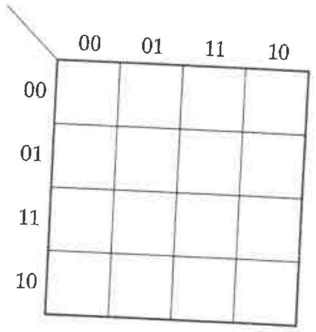
+ (1, 9) (0, 1, 8, 9)

(h) Using a map, find *all* of the prime implicants of Equation (6-2) and compare your answer with Equation (6-3).



(i) The prime implicants of $f(a, b, c, d) = \Sigma\, m(4, 5, 6, 7, 12, 13, 14, 15)$ are to be found using the Quine-McCluskey method. Column III is given; find Column IV and check off the appropriate terms in Column III.

	Column III	Column IV
(4, 5, 6, 7)	01 --	
(4, 5, 12, 13)	–10–	
(4, 6, 12, 14)	–1–0	
(5, 7, 13, 15)	–1–1	
(6, 7, 14, 15)	–11–	
(12, 13, 14, 15)	11 --	



Check your answer using a Karnaugh map.

3. (a) List all seven product term implicants of $F(a, b, c) = \Sigma\, m(0, 1, 5, 7)$

Which of these implicants are prime?

Why is $a'c$ not an implicant?

(b) Define a prime implicant.

(c) Why must every term in a minimum sum-of-products expression be a prime implicant?

(d) Given that $F(A, B, C, D) = \Sigma\, m(0, 1, 4, 5, 7, 10, 15)$, which of the following terms are *not* prime implicants and why?

$A'B'C'$ $A'C'$ BCD ABC $AB'CD'$

4. Study Section 6.2, *The Prime Implicant Chart.*

(a) Define an *essential* prime implicant.

(b) Find all of the essential prime implicants from the following chart.

	a b c d	0	4	5	10	11	12	13	15
(0, 4)	0 – 0 0	×	×						
(4, 5, 12, 13)	– 1 0 –		×	×			×	×	
(13, 15)	1 1 – 1							×	×
(11, 15)	1 – 1 1					×			×
(10, 11)	1 0 1 –				×	×			

Check your answer using a Karnaugh map.

(c) Why must all essential prime implicants of a function be included in the minimum sum of products?

(d) Complete the solution of Table 6-5.
(e) Work Programmed Exercise 6.1.
(f) Work Problems 6.2 and 6.3.

5. Study Section 6.3, *Petrick's Method* (optional).

(a) Consider the following reduced prime implicant chart for a function F:

		m_4	m_5	m_7	m_{13}
P_1	bd		×	×	×
P_2	bc'	×	×		×
P_3	$a'b$	×	×	×	
P_4	$c'd$		×		×

We will find all minimum solutions using Petrick's method. Let $P_i = 1$ mean the prime implicant in row P_i is included in the solution.
Which minterm is covered iff $(P_1 + P_3) = 1$?________
Write a sum term which is 1 iff m_4 is covered.________

Write a produc
covered:
$P =$ ________
(b) Reduce P to a
terms, each one
$P =$ ________
If $P_1P_2 = 1$, whi
How many min
Write out each

(1) $F =$

(3) $F =$

6. Study Section 6.4, *Sir*

(a) Why are don't-c
prime implicant

(b) Why are the do
chart when findi

(c) Work Problem 6
(d) Work Problem 6

7. If you have *LogicAid*
your answers to some
functions in the form
finds simplified sum-o
tions using a modified
It can also find one or

8. Study Section 6.5, *Sim*

(a) For the following
F is minimum by

BC \ A	0	1
00	D	1
01		
11	1	D
10	1	X

$m(0, 1, 4, 5, 7, 10, 15)$, which of the following
ts and why?

BCD $\quad$ ABC $\quad$ $AB'CD'$

icant Chart.

plicant.

e implicants from the following chart.

d	0	4	5	10	11	12	13	15
0	×	×						
–		×	×			×	×	
1							×	×
1					×			×
–				×	×			

arnaugh map.

e implicants of a function be included in the

le 6-5.
6.1.

d (optional).

ed prime implicant chart for a function F:

m_4	m_5	m_7	m_{13}
	×	×	×
×	×		×
×	×	×	
	×		×

itions using Petrick's method. Let $P_i = 1$ mean
$_i$ is included in the solution.
f $(P_1 + P_3) = 1$?________
iff m_4 is covered.________

Write a product-of-sum terms which is 1 iff all m_4, m_5, m_7 and m_{13} are all covered:

$P =$ ______________________________

(b) Reduce P to a minimum sum of products. (Your answer should have four terms, each one of the form P_iP_j.)

$P =$ ______________________________

If $P_1P_2 = 1$, which prime implicants are included in the solution?________
How many minimum solutions are there?________
Write out each solution in terms of a, b, c, and d.

(1) $F =$ $\qquad$ (2) $F =$

(3) $F =$ $\qquad$ (4) $F =$

6. Study Section 6.4, *Simplification of Incompletely Specified Functions.*

(a) Why are don't-care terms treated like required minterms when finding the prime implicants?

(b) Why are the don't-care terms not listed at the top of the prime implicant chart when finding the minimum solution?

(c) Work Problem 6.4.

(d) Work Problem 6.5, and check your solution using a Karnaugh map.

7. If you have *LogicAid* or a similar computer program available, use it to check your answers to some of the problems in this unit. *LogicAid* accepts Boolean functions in the form of equations, minterms or maxterms, and truth tables. It finds simplified sum-of-products and product-of-sums expressions for the functions using a modified version of the Quine-McCluskey method or Espresso-II. It can also find one or all of the minimum solutions using Petrick's method.

8. Study Section 6.5, *Simplification Using Map-Entered Variables.*

(a) For the following map, find MS_0, MS_1, and F. Verify that your solution for F is minimum by using a four-variable map.

BC \ A	0	1
00	*D*	1
01		
11	1	*D*
10	1	X

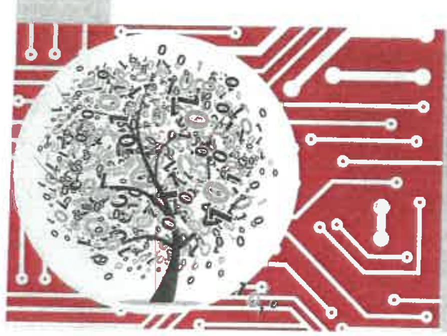
(b) Use the method of map-entered variables to find an expression for F from the following map. Treat C and C' as if they were independent variables. Is the result a correct representation of F? Is it minimum?

$B \backslash A$	0	1
0		C
1	C'	1

(c) Work Problem 6.6.

9. In this unit you have learned a "turn-the-crank" type procedure for finding minimum sum-of-products forms for switching functions. In addition to learning how to "turn the crank" and grind out minimum solutions, you should have learned several very important concepts in this unit. In particular, make sure you know:
   (a) What a prime implicant is
   (b) What an essential prime implicant is
   (c) Why the minimum sum-of-products form is a sum of prime implicants
   (d) How don't-cares are handled when using the Quine-McCluskey method and the prime implicant chart

10. Reread the objectives of the unit. If you are satisfied that you can meet the objectives, take the readiness test.



Quine-McCluskey Method

The Karnaugh map method described in Unit 5 is an effective way to simplify switching functions which have a small number of variables. When the number of variables is large or if several functions must be simplified, the use of a digital computer is desirable. The Quine-McCluskey method presented in this unit provides a systematic simplification procedure which can be readily programmed for a digital

The Quine-McCluske
of-products form) of a fun
consists of two main steps

1. Eliminate as many lit
the theorem $XY + X$
2. Use a prime implican
when ORed together,
tain a minimum numb

6.1 Determination

In order to apply the Qui
products expression for a f
(If the function is not in
using one of the techniqu
McCluskey method, all o
formed by combining min
and combined using

where X represents a produ
combine if they differ in ex
In order to find all of th
be compared and combine
comparisons, the binary mi
1's in each term. Thus,

$f(a, b, c$

is represented by the follow

The Quine-McCluskey method reduces the minterm expansion (standard sum-of-products form) of a function to obtain a minimum sum of products. The procedure consists of two main steps:

1. Eliminate as many literals as possible from each term by systematically applying the theorem $XY + XY' = X$. The resulting terms are called prime implicants.
2. Use a prime implicant chart to select a minimum set of prime implicants which, when ORed together, are equal to the function being simplified and which contain a minimum number of literals.

6.1 Determination of Prime Implicants

In order to apply the Quine-McCluskey method to determine a minimum sum-of-products expression for a function, the function must be given as a sum of minterms. (If the function is not in minterm form, the minterm expansion can be found by using one of the techniques given in Section 5.3.) In the first part of the Quine-McCluskey method, all of the prime implicants of a function are systematically formed by combining minterms. The minterms are represented in binary notation and combined using

$$XY + XY' = X \tag{6-1}$$

where X represents a product of literals and Y is a single variable. Two minterms will combine if they differ in exactly one variable.

In order to find all of the prime implicants, all possible pairs of minterms should be compared and combined whenever possible. To reduce the required number of comparisons, the binary minterms are sorted into groups according to the number of 1's in each term. Thus,

$$f(a, b, c, d) = \Sigma\, m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14) \tag{6-2}$$

is represented by the following list of minterms:

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

In this list, the term in group 0 has zero 1's, the terms in group 1 have one 1, those in group 2 have two 1's, and those in group 3 have three 1's.

Two terms can be combined if they differ in exactly one variable. Comparison of terms in nonadjacent groups is unnecessary because such terms will always differ in at least two variables and cannot be combined using $XY + XY' = X$. Similarly, the comparison of terms within a group is unnecessary because two terms with the same number of 1's must differ in at least two variables. Thus, only terms in adjacent groups must be compared.

First, we will compare the term in group 0 with all of the terms in group 1. Terms 0000 and 0001 can be combined to eliminate the fourth variable, which yields 000–. Similarly, 0 and 2 combine to form 00–0 ($a'b'd'$), and 0 and 8 combine to form –000 ($b'c'd'$). The resulting terms are listed in Column II of Table 6-1.

Whenever two terms combine, the corresponding decimal numbers differ by a power of 2 (1, 2, 4, 8, etc.). This is true because when the binary representations differ in exactly one column and if we subtract these binary representations, we get a 1 only in the column in which the difference exists. A binary number with a 1 in exactly one column is a power of 2.

TABLE 6-1 Determination of Prime Implicants

	Column I	Column II	Column III
group 0	0 0000 ✓	0, 1 000– ✓	0, 1, 8, 9 –00–
group 1	1 0001 ✓	0, 2 00–0 ✓	0, 2, 8, 10 –0–0
	2 0010 ✓	0, 8 –000 ✓	~~0, 8, 1, 9 –00–~~
	8 1000 ✓	1, 5 0–01	~~0, 8, 2, 10 –0–0~~
group 2	5 0101 ✓	1, 9 –001 ✓	2, 6, 10, 14 – –10
	6 0110 ✓	2, 6 0–10 ✓	~~2, 10, 6, 14 – –10~~
	9 1001 ✓	2, 10 –010 ✓	
	10 1010 ✓	8, 9 100– ✓	
group 3	7 0111 ✓	8, 10 10–0 ✓	
	14 1110 ✓	5, 7 01–1	
		6, 7 011–	
		6, 14 –110 ✓	
		10, 14 1–10 ✓	

Because the comparison of group 0 with groups 2 and 3 is unnecessary, we proceed to compare terms in groups 1 and 2. Comparing term 1 with all terms in group 2, we find that it combines with 5 and 9 but not with 6 or 10. Similarly, term 2 combines only with 6 and 10, and term 8 only with 9 and 10. The resulting terms are listed in Column II. Each time a term is combined with another term, it is checked off. A term may be used more than once because $X + X = X$. Even though two terms have already been combined with other terms, they still must be compared and combined if possible. This is necessary because the resultant term may be needed to form the

minimum sum solutic
redundant terms will
terms in groups 2 and
6 and 14, and 10 and 1
Note that the tern
the number of 1's in e
terms in Column II. I
variables, and the term
sary only to compare t
places and which diffe
Terms in the first
the second group whi
bines only with term
$a'b'c + ab'c' = b'c'$. Th
nation 0, 1, 8, 9 to indi
Term (0, 2) combines o
(2, 10). Again, the ter
terms from the second
with (10, 14), and (2, 10
Note that there are
terms were formed in
different order. After d
groups in Column III. B
nates. In general, we wo
and new columns until
The terms which ha
with other terms are c
included in at least one
its prime implicants. In t

$$f = a'c'd + a'bd$$
(1, 5) (5, 7

In this expression, e
ber of terms is not mini
terms yields

which is the minimum s
better method of elimi
cant chart.
Next, we will define i
Quine-McCluskey metho

minimum sum solution. At this stage, we may generate redundant terms, but these redundant terms will be eliminated later. We finish with Column I by comparing terms in groups 2 and 3. New terms are formed by combining terms 5 and 7, 6 and 7, 6 and 14, and 10 and 14.

Note that the terms in Column II have been divided into groups, according to the number of 1's in each term. Again, we apply $XY + XY' = X$ to combine pairs of terms in Column II. In order to combine two terms, the terms must have the same variables, and the terms must differ in exactly one of these variables. Thus, it is necessary only to compare terms which have dashes (missing variables) in corresponding places and which differ by exactly one in the number of 1's.

Terms in the first group in Column II need only be compared with terms in the second group which have dashes in the same places. Term 000– (0, 1) combines only with term 100– (8, 9) to yield –00–. This is algebraically equivalent to $a'b'c + ab'c' = b'c'$. The resulting term is listed in Column III along with the designation 0, 1, 8, 9 to indicate that it was formed by combining minterms 0, 1, 8, and 9. Term (0, 2) combines only with (8, 10), and term (0, 8) combines with both (1, 9) and (2, 10). Again, the terms which have been combined are checked off. Comparing terms from the second and third groups in Column II, we find that (2, 6) combines with (10, 14), and (2, 10) combines with (6, 14).

Note that there are three pairs of duplicate terms in Column III. These duplicate terms were formed in each case by combining the same set of four minterms in a different order. After deleting the duplicate terms, we compare terms from the two groups in Column III. Because no further combination is possible, the process terminates. In general, we would keep comparing terms and forming new groups of terms and new columns until no more terms could be combined.

The terms which have not been checked off because they cannot be combined with other terms are called prime implicants. Because every minterm has been included in at least one of the prime implicants, the function is equal to the sum of its prime implicants. In this example we have

$$f = \underset{(1,5)}{a'c'd} + \underset{(5,7)}{a'bd} + \underset{(6,7)}{a'bc} + \underset{(0,1,8,9)}{b'c'} + \underset{(0,2,8,10)}{b'd'} + \underset{(2,6,10,14)}{cd'} \tag{6-3}$$

In this expression, each term has a minimum number of literals, but the number of terms is not minimum. Using the consensus theorem to eliminate redundant terms yields

$$f = a'bd + b'c' + cd' \tag{6-4}$$

which is the minimum sum-of-products expression for f. Section 6.2 discusses a better method of eliminating redundant prime implicants using a prime implicant chart.

Next, we will define implicant and prime implicant and relate these terms to the Quine-McCluskey method.

Definition Given a function F of n variables, a product term P is an *implicant* of F iff for every combination of values of the n variables for which $P = 1$, F is also equal to 1.

In other words, if for some combination of values of the variables, $P = 1$ and $F = 0$, then P is *not* an implicant of F. For example, consider the function

$$F(a, b, c) = a'b'c' + ab'c' + ab'c + abc = b'c' + ac \tag{6-5}$$

If $a'b'c' = 1$, then $F = 1$; if $ac = 1$, then $F = 1$; etc. Hence, the terms $a'b'c'$, ac, etc., are implicants of F. In this example, bc is *not* an implicant of F because when $a = 0$ and $b = c = 1$, $bc = 1$ and $F = 0$. In general, if F is written in sum-of-products form, every product term is an implicant. Every minterm of F is also an implicant of F, and so is any term formed by combining two or more minterms. For example, in Table 6-1, all of the terms listed in any of the columns are implicants of the function given in Equation (6-2).

Definition A *prime implicant* of a function F is a product term implicant which is no longer an implicant if any literal is deleted from it.

In Equation (6-5), the implicant $a'b'c'$ *is not* a *prime* implicant because a' can be eliminated, and the resulting term ($b'c'$) is still an implicant of F. The implicants $b'c'$ and ac are *prime implicants* because *if* we delete a literal from either term, the term will no longer be an implicant of F. Each prime implicant of a function has a minimum number of literals in the sense that no more literals can be eliminated from it by combining it with other terms.

The Quine-McCluskey method, as previously illustrated, finds all of the product term implicants of a function. The implicants which are nonprime are checked off in the process of combining terms so that the remaining terms are prime implicants.

A minimum sum-of-products expression for a function consists of a sum of some (but not necessarily all) of the prime implicants of that function. In other words, a sum-of-products expression which contains a term which is not a prime implicant cannot be minimum. This is true because the nonprime term does not contain a minimum number of literals—it can be combined with additional minterms to form a prime implicant which has fewer literals than the nonprime term. Any nonprime term in a sum-of-products expression can thus be replaced with a prime implicant, which reduces the number of literals and simplifies the expression.

6.2 The Prime Implicant Chart

Given all the prime implicants of a function, the prime implicant chart can be used to select a minimum set of prime implicants. The minterms of the function are listed across the top of the chart, and the prime implicants are listed down the side.

product term P is an *implicant* of F iff for every
bles for which $P = 1$, F is also equal to 1.

tion of values of the variables, $P = 1$ and $F = 0$,
xample, consider the function

$$ab'c' + ab'c + abc = b'c' + ac \qquad (6\text{-}5)$$

n $F = 1$; etc. Hence, the terms $a'b'c'$, ac, etc., are
s *not* an implicant of F because when $a = 0$ and
al, if F is written in sum-of-products form, every
minterm of F is also an implicant of F, and so
o or more minterms. For example, in Table 6-1,
columns are implicants of the function given in

a product term implicant which is no longer an
m it.

$a'b'c'$ *is not* a *prime* implicant because a' can be
$'c'$) is still an implicant of F. The implicants $b'c'$
if we delete a literal from either term, the term
Each prime implicant of a function has a mini-
that no more literals can be eliminated from it

as previously illustrated, finds all of the product
implicants which are nonprime are checked off
that the remaining terms are prime implicants.
pression for a function consists of a sum of some
ie implicants of that function. In other words, a
contains a term which is not a prime implicant
ecause the nonprime term does not contain a
be combined with additional minterms to form
literals than the nonprime term. Any nonprime
on can thus be replaced with a prime implicant,
s and simplifies the expression.

t Chart

function, the prime implicant chart can be used
implicants. The minterms of the function are
d the prime implicants are listed down the side.

A prime implicant is equal to a sum of minterms, and the prime implicant is said to cover these minterms. If a prime implicant covers a given minterm, an X is placed at the intersection of the corresponding row and column. Table 6-2 shows the prime implicant chart derived from Table 6-1. All of the prime implicants (terms which have not been checked off in Table 6-1) are listed on the left.

In the first row, X's are placed in columns 0, 1, 8, and 9, because prime implicant $b'c'$ was formed from the sum of minterms 0, 1, 8, and 9. Similarly, X's are placed in columns 0, 2, 8, and 10 opposite the prime implicant $b'd'$ and so forth.

TABLE 6-2
Prime Implicant Chart

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	×	×					×	⊗		
(0, 2, 8, 10)	$b'd'$	×		×				×		×	
(2, 6, 10, 14)	cd'			×		×				×	⊗
(1, 5)	$a'c'd$		×		×						
(5, 7)	$a'bd$				×		×				
(6, 7)	$a'bc$					×	×				

If a minterm is covered by only one prime implicant, then that prime implicant is called an *essential* prime implicant and must be included in the minimum sum of products. Essential prime implicants are easy to find using the prime implicant chart. If a given column contains only one X, then the corresponding row is an essential prime implicant. In Table 6-2, columns 9 and 14 each contain one X, so prime implicants $b'c'$ and cd' are essential.

Each time a prime implicant is selected for inclusion in the minimum sum, the corresponding row should be crossed out. After doing this, the columns which correspond to all minterms covered by that prime implicant should also be crossed out. Table 6-3 shows the resulting chart when the essential prime implicants and the corresponding rows and columns of Table 6-2 are crossed out. A minimum set of prime implicants must now be chosen to cover the remaining columns. In this example, $a'bd$ covers the remaining two columns, so it is chosen. The resulting minimum sum of products is

$$f = b'c' + cd' + a'bd$$

which is the same as Equation (6-4). Note that even though the term $a'bd$ is included in the minimum sum of products, $a'bd$ is *not* an *essential* prime implicant. It is the sum of minterms m_5 and m_7; m_5 is also covered by $a'c'd$, and m_7 is also covered by $a'bc$.

TABLE 6-3

		0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	$b'c'$	×	×					×	×		
(0, 2, 8, 10)	$b'd'$	×		×				×		×	
(2, 6, 10, 14)	cd'			×		×				×	×
(1, 5)	$a'c'd$		×		×						
(5, 7)	$a'bd$				×		×				
(6, 7)	$a'bc$					×	×				

When selecting prime implicants for a minimum sum, the essential prime implicants are chosen first because all essential prime implicants must be included in every minimum sum. After the essential prime implicants have been chosen, the minterms which they cover can be eliminated from the prime implicant chart by crossing out the corresponding columns. If the essential prime implicants do not cover all of the minterms, then additional nonessential prime implicants are needed. In simple cases, the nonessential prime implicants needed to form the minimum solution may be selected by trial and error. For larger prime implicant charts, additional procedures for chart reduction can be employed.[1] (Also, see Problem 6.21.) Some functions have two or more minimum sum-of-products expressions, each having the same number of terms and literals. The next example shows such a function.

Example

A prime implicant chart which has two or more X's in every column is called a *cyclic* prime implicant chart. The following function has such a chart:

$$F = \Sigma\, m(0, 1, 2, 5, 6, 7) \tag{6-6}$$

Derivation of prime implicants:

0	000	✓		0, 1	00–
1	001	✓		0, 2	0–0
2	010	✓		1, 5	–01
5	101	✓		2, 6	–10
6	110	✓		5, 7	1–1
7	111	✓		6, 7	11–

Table 6-4 shows the resulting prime implicant chart. All columns have two X's, so we will proceed by trial and error. Both (0, 1) and (0, 2) cover column 0, so we will try (0, 1). After crossing out row (0, 1) and columns 0 and 1, we examine column 2, which is covered by (0, 2) and (2, 6). The best choice is (2, 6) because it covers two of the remaining columns while (0, 2) covers only one of the remaining columns. After crossing out row (2, 6) and columns 2 and 6, we see that (5, 7) covers the remaining columns and completes the solution. Therefore, one solution is $F = a'b' + bc' + ac$.

TABLE 6-4

			0	1	2	5	6	7
① →	(0, 1)	$a'b'$	X	X				
	(0, 2)	$a'c'$	X		X			
	(1, 5)	$b'c$		X		X		
② →	(2, 6)	bc'			X		X	
③ →	(5, 7)	ac				X		X
	(6, 7)	ab					X	X

[1][illegible]

However, we are not gu
and solve the problem over
column 0. The resulting tab

TABLE 6-5

			0	1
P_1	(0, 1)	$a'b'$	X	X
P_2	(0, 2)	$a'c'$	X	
P_3	(1, 5)	$b'c$		X
P_4	(2 6)	bc'		
P_5	(5, 7)	ac		
P_6	(6, 7)	ab		

Finish the solution and sh
number of terms and same
Table 6-4, there are two mi
pare these two minimum s
in Figure 5-9 using Karnau
covered by two different lo
(Table 6-4) has two X's, indi
prime implicants.

6.3 Petrick's Metho

Petrick's method is a techni
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two minimum solutions. As
implicants and the complexi
In such cases, a large amoun
solution(s). Petrick's metho
tions from a prime implicant
Petrick's method, all essenti
be removed from the chart.
We will illustrate Petrick
of the table P_1, P_2, P_3, etc. W
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when the prime implicant in
which is true when the prin
Because column 0 has X's in
cover minterm 0. Therefore,

However, we are not guaranteed that this solution is minimum. We must go back and solve the problem over again starting with the other prime implicant that covers column 0. The resulting table (Table 6-5) is

TABLE 6-5

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Finish the solution and show that $F = a'c' + b'c + ab$. Because this has the same number of terms and same number of literals as the expression for F derived in Table 6-4, there are two minimum sum-of-products solutions to this problem. Compare these two minimum solutions for Equation (6-6) with the solutions obtained in Figure 5-9 using Karnaugh maps. Note that each minterm on the map can be covered by two different loops. Similarly, each column of the prime implicant chart (Table 6-4) has two X's, indicating that each minterm can be covered by two different prime implicants.

6.3 Petrick's Method

Petrick's method is a technique for determining all minimum sum-of-products solutions from a prime implicant chart. The example shown in Tables 6-4 and 6-5 has two minimum solutions. As the number of variables increases, the number of prime implicants and the complexity of the prime implicant chart may increase significantly. In such cases, a large amount of trial and error may be required to find the minimum solution(s). Petrick's method is a more systematic way of finding all minimum solutions from a prime implicant chart than the method used previously. Before applying Petrick's method, all essential prime implicants and the minterms they cover should be removed from the chart.

We will illustrate Petrick's method using Table 6-5. First, we will label the rows of the table P_1, P_2, P_3, etc. We will form a logic function, P, which is true when all of the minterms in the chart have been covered. Let P_1 be a logic variable which is true when the prime implicant in row P_1 is included in the solution, P_2 be a logic variable which is true when the prime implicant in row P_2 is included in the solution, etc. Because column 0 has X's in rows P_1 and P_2, we must choose row P_1 or P_2 in order to cover minterm 0. Therefore, the expression $(P_1 + P_2)$ must be true. In order to cover minterm 1, we must choose row P_1 or P_3; therefore, $(P_1 + P_3)$ must be true. In order to cover minterm 2, $(P_2 + P_4)$ must be true. Similarly, in order to cover minterms 5, 6, and 7, the expressions $(P_3 + P_5)$, $(P_4 + P_6)$ and $(P_5 + P_6)$ must be true. Because we must cover all of the minterms, the following function must be true:

$$P = (P_1 + P_2)(P_1 + P_3)(P_2 + P_4)(P_3 + P_5)(P_4 + P_6)(P_5 + P_6) = 1$$

The expression for P in effect means that we must choose row P_1 or P_2, *and* row P_1 or P_3, *and* row P_2 or P_4, etc.

The next step is to reduce P to a minimum sum of products. This is easy because there are no complements. First, we multiply out, using $(X + Y)(X + Z) = X + YZ$ and the ordinary distributive law:

$$\begin{aligned} P &= (P_1 + P_2P_3)(P_4 + P_2P_6)(P_5 + P_3P_6) \\ &= (P_1P_4 + P_1P_2P_6 + P_2P_3P_4 + P_2P_3P_6)(P_5 + P_3P_6) \\ &= P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_2P_3P_5P_6 + P_1P_3P_4P_6 \\ &\quad + P_1P_2P_3P_6 + P_2P_3P_4P_6 + P_2P_3P_6 \end{aligned}$$

Next, we use $X + XY = X$ to eliminate redundant terms from P, which yields

$$P = P_1P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_4P_5 + P_1P_3P_4P_6 + P_2P_3P_6$$

Because P must be true ($P = 1$) in order to cover all of the minterms, we can translate the equation back into words as follows. In order to cover all of the minterms, we must choose rows P_1 and P_4 and P_5, or rows P_1 and P_2 and P_5 and P_6, or . . . or rows P_2 and P_3 and P_6. Although there are five possible solutions, only two of these have the minimum number of rows. Thus, the two solutions with the minimum number of prime implicants are obtained by choosing rows P_1, P_4, and P_5 or rows P_2, P_3, and P_6. The first choice leads to $F = a'b' + bc' + ac$, and the second choice to $F = a'c' + b'c + ab$, which are the two minimum solutions derived in Section 6.2.

In summary, Petrick's method is as follows:

1. Reduce the prime implicant chart by eliminating the essential prime implicant rows and the corresponding columns.
2. Label the rows of the reduced prime implicant chart P_1, P_2, P_3, etc.
3. Form a logic function P which is true when all columns are covered. P consists of a product of sum terms, each sum term having the form $(P_{i0} + P_{i1} + \cdots)$, where $P_{i0}, P_{i1} \ldots$ represent the rows which cover column i.
4. Reduce P to a minimum sum of products by multiplying out and applying $X + XY = X$.
5. Each term in the result represents a solution, that is, a set of rows which covers all of the minterms in the table. To determine the minimum solutions (as defined in Section 5.1), find those terms which contain a minimum number of variables. Each of these terms represents a solution with a minimum number of prime implicants.
6. For each of the terms found in step 5, count the number of literals in each prime implicant and find the total number of literals. Choose the term or terms which correspond to the minimum total number of literals, and write out the corresponding sums of prime implicants.

The application of Petri
implement on a comput

6.4 Simplification Specified Fun

Given an incompletely s
don't-care terms is neces
In this section, we will sh
to obtain a minimum so
of finding the prime imp
required minterms. In this
nate as many literals as p
of the don't-cares, this is
nated in the next step any
cares are *not* listed at the t
of the required minterms
However, the don't-care
have been used in the pro
following example of simp
the procedure.

$F(A, B, C, D) =$

The don't-care terms are t
implicants:

1	0001	✓
2	0010	✓
3	0011	✓
9	1001	✓
10	1010	✓
7	0111	✓
11	1011	✓
13	1101	✓
5	1111	✓

The application of Petrick's method is very tedious for large charts, but it is easy to implement on a computer.

6.4 Simplification of Incompletely Specified Functions

Given an incompletely specified function, the proper assignment of values to the don't-care terms is necessary in order to obtain a minimum form for the function. In this section, we will show how to modify the Quine-McCluskey method in order to obtain a minimum solution when don't-care terms are present. In the process of finding the prime implicants, we will treat the don't-care terms as if they were required minterms. In this way, they can be combined with other minterms to eliminate as many literals as possible. If extra prime implicants are generated because of the don't-cares, this is correct because the extra prime implicants will be eliminated in the next step anyway. When forming the prime implicant chart, the don't-cares are *not* listed at the top. This way, when the prime implicant chart is solved, all of the required minterms will be covered by one of the selected prime implicants. However, the don't-care terms are not included in the final solution unless they have been used in the process of forming one of the selected prime implicants. The following example of simplifying an incompletely specified function should clarify the procedure.

$$F(A, B, C, D) = \Sigma\, m(2, 3, 7, 9, 11, 13) + \Sigma\, d(1, 10, 15)$$

(the terms following d are don't-care terms)

The don't-care terms are treated like required minterms when finding the prime implicants:

1	0001 ✓	(1, 3)	00–1 ✓	(1, 3, 9, 11)	–0–1
2	0010 ✓	(1, 9)	–001 ✓	(2, 3, 10, 11)	–01–
3	0011 ✓	(2, 3)	001– ✓	(3, 7, 11, 15)	- -11
9	1001 ✓	(2, 10)	–010 ✓	(9, 11, 13, 15)	1- -1
10	1010 ✓	(3, 7)	0–11 ✓		
7	0111 ✓	(3, 11)	–011 ✓		
11	1011 ✓	(9, 11)	10–1 ✓		
13	1101 ✓	(9, 13)	1–01 ✓		
5	1111 ✓	(10, 11)	101– ✓		
		(7, 15)	–111 ✓		
		(11, 15)	1–11 ✓		
		(13, 15)	11–1 ✓		

The don't-care columns are omitted when forming the prime implicant chart:

	2	3	7	9	11	13
(1, 3, 9, 11)		×		×	×	
*(2, 3, 10, 11)	×	×			×	
*(3, 7, 11, 15)		×	×		×	
*(9, 11, 13, 15)				×	×	×

*Indicates an essential prime implicant.

$$F = B'C + CD + AD$$

Note that although the original function was incompletely specified, the final simplified expression for F is defined for all combinations of values for $A, B, C,$ and D and is therefore completely specified. In the process of simplification, we have automatically assigned values to the don't-cares in the original truth table for F. If we replace each term in the final expression for F by its corresponding sum of minterms, the result is

$$F = (m_2 + m_3 + m_{10} + m_{11}) + (\cancel{m_3} + m_7 + \cancel{m_{11}} + m_{15}) + (m_9 + \cancel{m_{11}} + m_{13} + \cancel{m_{15}})$$

Because m_{10} and m_{15} appear in this expression and m_1 does not, this implies that the don't-care terms in the original truth table for F have been assigned as follows:

for $ABCD = 0001, F = 0$; for $1010, F = 1$; for $1111, F = 1$

6.5 Simplification Using Map-Entered Variables

Although the Quine-McCluskey method can be used with functions with a fairly large number of variables, it is not very efficient for functions that have many variables and relatively few terms. Some of these functions can be simplified by using a modification of the Karnaugh map method. By using map-entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables. Figure 6-1(a) shows a four-variable map with two additional variables entered in the squares in the map. When E appears in a square, this means

FIGURE 6-1 Use of Map-Entered Variables

CD \ AB	00	01	11	10
00	1			
01	X	E	X	F
11	1	E	1	1
10	1			X

G

CD \ AB	00	01	11	10
00	1			
01	X		X	
11	1		1	1
10	1			X

$E = F = 0$
$MS_0 = A'B' + ACD$

CD \ AB	00	01	11	10
00	X			
01	X	1	X	
11	X	1	X	X
10	X			X

$E = 1, F = 0$
$MS_1 = A'D$

CD \ AB	00	01	11	10
00	X			
01	X		X	1
11	X		X	X
10	X			X

$E = 0, F = 1$
$MS_2 = AD$

that if $E = 1$, the corr…
the minterm is absent.

$G(A, B, C, D, E$…

where the minterms a…
present in G only whe…
We will now use a …

$F(A, B, C, D$…

where the $AB'C$ is a d…
choose it as a map-ent…
by first considering D =…
to $A'C$. Setting $D = 1$ l…
map have already been…
we do not care wheth…
$D = 1$. Thus, the expres…

$F =$ …

gives the correct value…
expression for F, as can…
map; see Figure 6-2(c).
Next, we will discuss…
variables. In general, if a…
means that $F = 1$ when…
map with variables P_1, P…
products form of F can…
Find a sum-of-produ…

where

MS_0 is the minimum…

FIGURE 6-2 Simplification Using a Map-Entered Variable

BC \ A	0	1
00		
01	1	X
11	1	D
10	D	

when forming the prime implicant chart:

$$F = B'C + CD + AD$$

ıt.

function was incompletely specified, the final ed for all combinations of values for $A, B, C,$ ›ecified. In the process of simplification, we have e don't-cares in the original truth table for F. l expression for F by its corresponding sum of

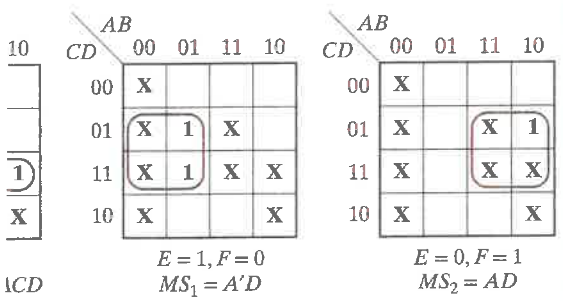
$$+ m_7 + m_{11} + m_{15}) + (m_9 + m_{11} + m_{13} + m_{15})$$

ᴈxpression and m_1 does not, this implies that the ı table for F have been assigned as follows:

), $F = 1$; for 1111, $F = 1$

ʒ Map-Entered Variables

ethod can be used with functions with a fairly very efficient for functions that have many vari- ıe of these functions can be simplified by using nap method. By using map-entered variables, extended to simplify functions with more than shows a four-variable map with two additional he map. When E appears in a square, this means



(c) (d)

that if $E = 1$, the corresponding minterm is present in the function G, and if $E = 0$, the minterm is absent. Thus, the map represents the six-variable function

$$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$$
$$(+\text{don't-care terms})$$

where the minterms are minterms of the variables A, B, C, and D. Note that m_9 is present in G only when $F = 1$.

We will now use a three-variable map to simplify the function:

$$F(A, B, C, D) = A'B'C + A'BC + A'BC'D + ABCD + (AB'C)$$

where the $AB'C$ is a don't-care term. Because D appears in only two terms, we will choose it as a map-entered variable, which leads to Figure 6-2(a). We will simplify F by first considering $D = 0$ and then $D = 1$. First set $D = 0$ on the map, and F reduces to $A'C$. Setting $D = 1$ leads to the map of Figure 6-2(b). The two 1's on the original map have already been covered by the term $A'C$, so they are changed to X's because we do not care whether they are covered again or not. From Figure 6-2(b), when $D = 1$. Thus, the expression

$$F = A'C + D(C + A'B) = A'C + CD + A'BD$$

gives the correct value of F both when $D = 0$ and when $D = 1$. This is a minimum expression for F, as can be verified by plotting the original function on a four-variable map; see Figure 6-2(c).

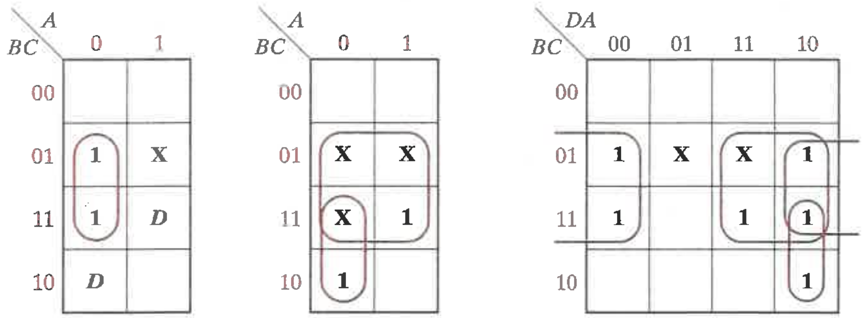
Next, we will discuss a general method of simplifying functions using map-entered variables. In general, if a variable P_i is placed in square m_j of a map of function F, this means that $F = 1$ when $P_i = 1$, and the variables are chosen so that $m_j = 1$. Given a map with variables $P_1, P_2, \ldots$ entered into some of the squares, the minimum sum-of-products form of F can be found as follows:

Find a sum-of-products expression for F of the form

$$F = MS_0 + P_1MS_1 + P_2MS_2 + \cdots$$

where

MS_0 is the minimum sum obtained by setting $P_1 = P_2 = \cdots = 0$.



(a) (b) (c)

FIGURE 6-2 Simplification Using a Map-Entered Variable

MS_1 is the minimum sum obtained by setting $P_1 = 1$, $P_j = 0\ (j \neq 1)$, and replacing all 1's on the map with don't-cares.
MS_2 is the minimum sum obtained by setting $P_2 = 1$, $P_j = 0\ (j \neq 2)$ and replacing all 1's on the map with don't-cares.

(Corresponding minimum sums can be found in a similar way for any remaining map-entered variables.)

The resulting expression for F will always be a correct representation of F. This expression will be minimum provided that the values of the map-entered variables can be assigned independently. On the other hand, the expression will not generally be minimum if the variables are not independent (for example, if $P_1 = P_2'$).

For the example of Figure 6-1(a), maps for finding MS_0, MS_1, and MS_2 are shown in Figures 6-1(b), (c), and (d), where E corresponds to P_1 and F corresponds to P_2. The resulting expression is a minimum sum of products for G:

$$G = A'B' + ACD + EA'D + FAD$$

After some practice, it should be possible to write the minimum expression directly from the original map without first plotting individual maps for each of the minimum sums.

6.6 Conclusion

We have discussed four methods for reducing a switching expression to a minimum sum-of-products or a minimum product-of-sums form: algebraic simplification, Karnaugh maps, Quine-McCluskey method, and Petrick's method. Many other methods of simplification are discussed in the literature, but most of these methods are based on variations or extensions of the Karnaugh map or Quine-McCluskey techniques. Karnaugh maps are most useful for functions with three to five variables. The Quine-McCluskey technique can be used with a high-speed digital computer to simplify functions with up to 15 or more variables. Such computer programs are of greatest value when used as part of a computer-aided design (CAD) package that assists with deriving the equations as well as implementing them. Algebraic simplification is still valuable in many cases, especially when different forms of the expressions are required. For problems with a large number of variables and a small number of terms, it may be impossible to use the Karnaugh map, and the Quine-McCluskey method may be very cumbersome. In such cases, algebraic simplification may be the easiest method to use. In situations where a minimum solution is not required or where obtaining a minimum solution requires too much computation to be practical, heuristic procedures may be used to simplify switching functions. One of the more popular heuristic procedures is the Espresso-II method,[2] which can produce near minimum solutions for a large class of problems.

The minimum sum-of-products and minimum product-of-sums expressions we have derived lead directly to two-level circuits that use a minimum number of AND and OR gates and have a
these circuits are easily tra
These minimum expressic
array logic, as discussed ir
expressions do not lead to
must be considered, such a

What is the maximum
What is the maximum
Is the speed with whic
How can the number
Does the design lead t
on a silicon chip?

Until now, we have co
Unit 7 describes design to
used when several functio

Programmed I

Cover the answers to this
check your answers.

Find a minimum sum

$f(A, B, C, D, E) = $

Translate each decimal mi
according to the number of

Answer:

0	00000 ✓		0,2	000
2	00010 ✓			
16	10000			
3	00011			
5	00101			
9	01001			
18	10010			
24	11000			
7	00111			
11	01011			
13	01101			
14	01110			
26	11010			
28	11100			
30	11110			

ed by setting $P_1 = 1, P_j = 0\ (j \neq 1)$, and replac-
on't-cares.
ed by setting $P_2 = 1, P_j = 0\ (j \neq 2)$ and replac-
on't-cares.

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d that the values of the map-entered variables
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$ACD + EA'D + FAD$

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ut first plotting individual maps for each of the

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f-sums form: algebraic simplification, Karnaugh
Petrick's method. Many other methods of sim-
re, but most of these methods are based on vari-
map or Quine-McCluskey techniques. Karnaugh
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where obtaining a minimum solution requires too
ıristic procedures may be used to simplify switch-
r heuristic procedures is the Espresso-II method,[2]
utions for a large class of problems.
and minimum product-of-sums expressions we
el circuits that use a minimum number of AND

et al., *Logic Minimization Algorithms for VLSI Synthesis*

and OR gates and have a minimum number of gate inputs. As discussed in Unit 7, these circuits are easily transformed into circuits that contain NAND or NOR gates. These minimum expressions may also be useful when designing with some types of array logic, as discussed in Unit 9. However, many situations exist where minimum expressions do not lead to the best design. For practical designs, many other factors must be considered, such as the following:

What is the maximum number of inputs a gate can have?
What is the maximum number of outputs a gate can drive?
Is the speed with which signals propagate through the circuit fast enough?
How can the number of interconnections in the circuit be reduced?
Does the design lead to a satisfactory circuit layout on a printed circuit board or on a silicon chip?

Until now, we have considered realizing only one switching function at a time. Unit 7 describes design techniques and Unit 9 describes components that can be used when several functions must be realized by a single circuit.

Programmed Exercise 6.1

Cover the answers to this exercise with a sheet of paper and slide it down as you check your answers.

Find a minimum sum-of-products expression for the following function:

$$f(A, B, C, D, E) = \Sigma\, m(0, 2, 3, 5, 7, 9, 11, 13, 14, 16, 18, 24, 26, 28, 30)$$

Translate each decimal minterm into binary and sort the binary terms into groups according to the number of 1's in each term.

Answer:

0	00000 ✓	0,2	000-0
2	00010 ✓		
16	10000		
3	00011		
5	00101		
9	01001		
18	10010		
24	11000		
7	00111		
11	01011		
13	01101		
14	01110		
26	11010		
28	11100		
30	11110		

Compare pairs of terms in adjacent groups and combine terms where possible. (Check off terms which have been combined.)

Answer:

0	00000 ✓	0, 2	000–0 ✓	0, 2, 16, 18	–00–0
2	00010 ✓	0, 16	–0000		
16	10000 ✓	2, 3	0001–		
3	00011 ✓	2, 18	–0010		
5	00101 ✓	16, 18	100–0 ✓		
9	01001 ✓	16, 24	1–000		
18	10010 ✓	3, 7	00–11		
24	11000 ✓	3, 11	0–011		
7	00111 ✓	5, 7	001–1		
11	01101 ✓	5, 13	0–101		
13	01101 ✓	9, 11	010–1		
14	01110 ✓	9, 13	01–01		
26	11010 ✓	18, 26	1–010		
28	11100 ✓	24, 26	110–0		
30	11110 ✓	24, 28	11–00		
		14, 30	–1110		
		26, 30	11–10		
		28, 30	111–0		

Now, compare pairs of terms in adjacent groups in the second column and combine terms where possible. (Check off terms which have been combined.) Check your work by noting that each new term can be formed in two ways. (Cross out duplicate terms.)

Answer: (third column)

0, 2, 16, 18	–00–0	(check off (0, 2), (16, 18), (0, 16), and (2, 18))
16, 18, 24, 26	1–0–0	(check off (16, 18), (24, 26), (16, 24), and (18, 26))
24, 26, 28, 30	11--0	(check off (24, 26), (28, 30), (24, 28), and (26, 30))

Can any pair of terms in the third column be combined?
Complete the given prime implicant chart.

	0	2
(0, 2, 16, 18)		

Answer: No pair of terms in the tl

	0	2	3
(0, 2, 16, 18)	X	X	
(16, 18, 24, 26)			
(24, 26, 28, 30)			
(2, 3)		X	X
(3, 7)			X
(3, 11)			X
(5, 7)			
(5, 13)			
(9, 11)			
(9, 13)			
(14, 30)			

Determine the essential pı
columns.

Answer:

	0	2	3	5
*(0, 2, 16, 18)	⊗	~~X~~		
(16, 18, 24, 26)				
*(24, 26, 28, 30)				
(2, 3)		~~X~~	X	
(3, 7)			X	›
(3, 11)			X	
(5, 7)				X ›
(5, 13)				X
(9, 11)				
(9, 13)				
*(14, 30)				

*Indicates an essential prime

Note that all remaining colı
which has two X's and then
that column. Then, choose a
remaining columns in the ch

Answer: No pair of terms in the third column combine.

	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
(0, 2, 16, 18)	×	×								×	×				
(16, 18, 24, 26)										×	×	×	×		
(24, 26, 28, 30)												×	×	×	×
(2, 3)		×	×												
(3, 7)			×		×										
(3, 11)			×				×								
(5, 7)				×	×										
(5, 13)				×				×							
(9, 11)						×	×								
(9, 13)						×		×							
(14, 30)									×						×

Determine the essential prime implicants, and cross out the corresponding rows and columns.

Answer:

	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
*(0, 2, 16, 18)	⊗	×								×	×				
(16, 18, 24, 26)										×	×	×	×		
*(24, 26, 28, 30)												×	×	⊗	×
(2, 3)		×	×												
(3, 7)			×		×										
(3, 11)			×				×								
(5, 7)				×	×										
(5, 13)				×				×							
(9, 11)						×	×								
(9, 13)						×		×							
*(14, 30)									⊗						×

*Indicates an essential prime implicant.

Note that all remaining columns contain two or more X's. Choose the first column which has two X's and then select the prime implicant which covers the first X in that column. Then, choose a minimum number of prime implicants which cover the remaining columns in the chart.

Answer:

	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
*(0, 2, 16, 18)	⊗	X								X	X				
(16, 18, 24, 26)										X	X	X	X		
*(24, 26, 28, 30)												X	X	⊗	X
(2, 3)		X	X												
(3, 7)			X		X										
→ (3, 11)			X				X								
→ (5, 7)				X	X										
(5, 13)				X				X							
(9, 11)						X	X								
→ (9, 13)						X		X							
*(14, 30)									⊗						X

*Indicates an essential prime implicant.

From this chart, write down the chosen prime implicants in 0, 1, and – notation.

Then, write the minimum sum of products in algebraic form.

Answer: –00–0, 11--0, 0–011, 001–1, 01–01, and –1110
$f = B'C'E' + ABE' + A'C'DE + A'B'CE + A'BD'E + BCDE'$

The prime implicant chart with the essential prime implicants crossed out is repeated here.
Find a second minimum sum-of-products solution.

	0	2	3	5	7	9	11	13	14	16	18	24	26	28	30
*(0, 2, 16, 18)	⊗	X								X	X				
(16, 18, 24, 26)										X	X	X	X		
*(24, 26, 28, 30)												X	X	⊗	X
(2, 3)		X	X												
(3, 7)			X		X										
(3, 11)			X				X								
(5, 7)				X	X										
(5, 13)				X				X							
(9, 11)						X	X								
(9, 13)						X		X							
*(14, 30)									⊗						X

*Indicates an essential prime implicant.

Answer: Start by choosing prime implicant (5, 13).
$f = BCDE' + B'C'E' + ABE' + A'B'DE + A'CD'E + A'BC'E$

Find the minimum sum-of-products expression for F, using the Quine-McCluskey method. Underline the essential prime implicants in this expression.

6.12 Using the Quine-McCluskey method, find all minimum sum-of-products expressions for
(a) $f(A, B, C, D, E) = \Sigma\, m(0, 1, 2, 3, 4, 8, 9, 10, 11, 19, 21, 22, 23, 27, 28, 29, 30)$
(b) $f(A, B, C, D, E) = \Sigma\, m(0, 1, 2, 4, 8, 11, 13, 14, 15, 17, 18, 20, 21, 26, 27, 30, 31)$

6.13 Using the Quine-McCluskey method, find all minimum product-of-sums expressions for the functions of Problem 6.12.

6.14 (a) Using the Quine-McCluskey, method find all prime implicants of $f(A, B, C, D) = \Sigma\, m(1, 3, 5, 6, 8, 9, 12, 14, 15) + \Sigma\, d(4, 10, 13)$. Identify all essential prime implicants and find all minimum sum-of-products expressions.
(b) Repeat part (a) for f'.

6.15 (a) Use the Quine-McCluskey method to find all prime implicants of $f(a, b, c, d, e) = \Sigma\, m(1, 2, 4, 5, 6, 7, 9, 12, 13, 15, 17, 20, 22, 25, 28, 30)$. Find all essential prime implicants, and find all minimum sum-of-products expressions.
(b) Repeat part (a) for f'.

6.16 $G(A, B, C, D, E, F) = \Sigma\, m(1, 2, 3, 16, 17, 18, 19, 26, 32, 39, 48, 63)$
$+ \Sigma\, d(15, 28, 29, 30)$
(a) Find all minimum sum-of-products expressions for G.
(b) Circle the *essential* prime implicants in your answer.
(c) If there were no don't-care terms present in the original function, how would your answer to part (a) change? (Do this by inspection of the prime implicant chart; do *not* rework the problem.)

6.17 (a) Use the Quine-McCluskey procedure to find *all* prime implicants of the function $G(A, B, C, D, E, F) = \Sigma\, m(1, 7, 11, 12, 15, 33, 35, 43, 47, 59, 60) + \Sigma\, d(30, 50, 54, 58)$. Identify all essential prime implicants and find all minimum sum-of-products expressions.
(b) Repeat part (a) for G'.

6.18 The following prime implicant table (chart) is for a four-variable function $f(A, B, C, D)$.
(a) Give the decimal representation for each of the prime implicants.
(b) List the maxterms of f.
(c) List the don't-cares of f, if any.
(d) Give the algebraic expression for each of the essential prime implicants.

	2	3	7	9	11	13
–0–1		×		×	×	
–01–	×	×			×	
- -11		×	×		×	

6.19 Packages arrive at the st
ries by student employe
students are paid accor
their use is
Cart C1: \$2
Cart C2: \$1
Cart C3: \$4
Cart C4: \$2
Cart C5: \$2
On a particular day, seve
carts as follows:
C1 can be used for packa
C2 can be used for packa
C3 can be used for packa
C4 can be used for packa
C5 can be used for packa

The stockroom manager w
mization techniques descr
ing the minimum cost solu

6.20 Use the Quine-McCluske
$h(A, B, C, D, E, F, G) = \Sigma$
Express the prime implica

6.21 Shown below is the prime
combinational logic functi
(a) Algebraically express r
(b) Give algebraic expressi
table.
(c) Find all minimal sum-of
braic expressions; inste
in the sum(s).

	0	4	
A	×	×	
B			
C			
D			
E			
F			
G	×		
H		×	×

16 18 24 26 28 30

t.

n prime implicants in 0, 1, and – notation.

products in algebraic form.

1 –1110

$'B'CE + A'BD'E + BCDE'$

sential prime implicants crossed out is repeated

lucts solution.

4 16 18 24 26 28 30

nt.

, 13).

$+ A'B'DE + A'CD'E + A'BC'E$

Problems

6.2 For each of the following functions, find all of the prime implicants, using the Quine-McCluskey method.

(a) $f(a, b, c, d) = \Sigma\, m(1, 5, 7, 9, 11, 12, 14, 15)$

(b) $f(a, b, c, d) = \Sigma\, m(0, 1, 3, 5, 6, 7, 8, 10, 14, 15)$

6.3 Using a prime implicant chart, find *all* minimum sum-of-products solutions for each of the functions given in Problem 6.2.

6.4 For this function, find a minimum sum-of-products solution, using the Quine-McCluskey method.

$$f(a, b, c, d) = \Sigma\, m(1, 3, 4, 5, 6, 7, 10, 12, 13) + \Sigma\, d(2, 9, 15)$$

6.5 Find all prime implicants of the following function and then find all minimum solutions using Petrick's method:

$$F(A, B, C, D) = \Sigma\, m(9, 12, 13, 15) + \Sigma\, d(1, 4, 5, 7, 8, 11, 14)$$

6.6 Using the method of map-entered variables, use four-variable maps to find a minimum sum-of-products expression for

(a) $F(A, B, C, D, E) = \Sigma\, m(0, 4, 5, 7, 9) + \Sigma\, d(6, 11) + E(m_1 + m_{15})$, where the m's represent minterms of the variables A, B, C, and D.

(b) $Z(A, B, C, D, E, F, G) = \Sigma\, m(0, 3, 13, 15) + \Sigma\, d(1, 2, 7, 9, 14) + E(m_6 + m_8) + Fm_{12} + Gm_5$

6.7 For each of the following functions, find all of the prime implicants using the Quine-McCluskey method.

(a) $f(a, b, c, d) = \Sigma\, m(0, 3, 4, 5, 7, 9, 11, 13)$

(b) $f(a, b, c, d) = \Sigma\, m(2, 4, 5, 6, 9, 10, 11, 12, 13, 15)$

6.8 Using a prime implicant chart, find *all* minimum sum-of-products solutions for each of the functions given in Problem 6.7.

6.9 For each function, find a minimum sum-of-products solution using the Quine-McCluskey method.

(a) $f(a, b, c, d) = \Sigma\, m(2, 3, 4, 7, 9, 11, 12, 13, 14) + \Sigma\, d(1, 10, 15)$

(b) $f(a, b, c, d) = \Sigma\, m(0, 1, 5, 6, 8, 9, 11, 13) + \Sigma\, d(7, 10, 12)$

(c) $f(a, b, c, d) = \Sigma\, m(3, 4, 6, 7, 8, 9, 11, 13, 14) + \Sigma\, d(2, 5, 15)$

6.10 Work Problem 5.24(a) using the Quine-McCluskey method.

6.11 $F(A, B, C, D, E) = \Sigma\, m(0, 2, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 19, 29, 30) + \Sigma\, d(4, 9, 21)$

s expression for F, using the Quine-McCluskey
me implicants in this expression.

ıod, find all minimum sum-of-products expres-

3, 4, 8, 9, 10, 11, 19, 21, 22, 23, 27, 28, 29, 30)
4, 8, 11, 13, 14, 15, 17, 18, 20, 21, 26, 27, 30, 31)

ıd, find all minimum product-of-sums expressions

ethod find all prime implicants of $f(A, B, C, D) =$
$\Sigma\, d(4, 10, 13)$. Identify all essential prime impli-
n-of-products expressions.

hod to find all prime implicants of $f(a, b, c, d, e) =$
17, 20, 22, 25, 28, 30). Find all essential prime
m sum-of-products expressions.

16, 17, 18, 19, 26, 32, 39, 48, 63)
, 28, 29, 30)
lucts expressions for G.
icants in your answer.
rms present in the original function, how would
e? (Do this by inspection of the prime implicant
em.)

procedure to find *all* prime implicants of the
$F) = \Sigma\, m(1, 7, 11, 12, 15, 33, 35, 43, 47, 59, 60) +$
essential prime implicants and find all minimum

(chart) is for a four-variable function $f(A, B, C, D)$.
on for each of the prime implicants.

7.

for each of the essential prime implicants.

3	7	9	11	13
×		×	×	
×			×	
×	×		×	
		×	×	×

6.19 Packages arrive at the stockroom and are delivered on carts to offices and laboratories by student employees. The carts and packages are various sizes and shapes. The students are paid according to the carts used. There are five carts and the pay for their use is
Cart C1: \$2
Cart C2: \$1
Cart C3: \$4
Cart C4: \$2
Cart C5: \$2
On a particular day, seven packages arrive, and they can be delivered using the five carts as follows:
C1 can be used for packages P1, P3, and P4.
C2 can be used for packages P2, P5, and P6.
C3 can be used for packages P1, P2, P5, P6, and P7.
C4 can be used for packages P3, P6, and P7.
C5 can be used for packages P2 and P4.

The stockroom manager wants the packages delivered at minimum cost. Using minimization techniques described in this unit, present a systematic procedure for finding the minimum cost solution.

6.20 Use the Quine-McCluskey procedure to find all prime implicants of the function $h(A, B, C, D, E, F, G) = \Sigma\, m(24, 28, 39, 47, 70, 86, 88, 92, 102, 105, 118)$. Express the prime implicants *algebraically*.

6.21 Shown below is the prime implicant chart for a completely specified four-variable combinational logic function $r(w, x, y, z)$.
(a) Algebraically express r as a product of maxterms.
(b) Give algebraic expressions for the prime implicants labeled A, C, and D in the table.
(c) Find all minimal sum-of-product expressions for r. You do **not** have to give algebraic expressions; instead just list the prime implicants (A, B, C, etc.) required in the sum(s).

	0	4	5	6	7	8	9	10	11	13	14	15
A	×	×										
B			×		×					×		×
C				×	×						×	×
D						×	×	×	×			
E								×	×		×	×
F							×		×	×		×
G	×					×						
H		×	×	×	×							

6.22 (a) In the prime implicant chart of Problem 6.21, column 7 is said to *cover* column 6 since column 7 has an X in each row that column 6 does. Similarly, column 11 covers column 10 and column 15 covers column 14. Columns 7, 11, and 15 can be removed to obtain a simpler chart having the same solutions as the original. Explain why this is correct.

(b) In Table 6-5 (after removing row P_2 and columns 0 and 2), row P_3 covers row P_1. Row (prime implicant) P_1 can be removed, and the resulting chart will have a minimum solution for the original table. Explain why this is correct. Are there any restrictions on the two prime implicants to allow removal of the covered prime implicant?

(c) After deleting row P_1 from Table 6-5, row P_3 must be included in a minimal solution for the chart. Why?

6.23 Find all prime implicants of the following function, and then find all minimum solutions using Petrick's method:

$$F(A, B, C, D) = \Sigma\, m(7, 12, 14, 15) + \Sigma\, d(1, 3, 5, 8, 10, 11, 13)$$

6.24 Using the method of map-entered variables, use four-variable maps to find a minimum sum-of-products expression for

(a) $F(A, B, C, D, E) = \Sigma\, m(0, 4, 6, 13, 14) + \Sigma\, d(2, 9) + E(m_1 + m_{12})$

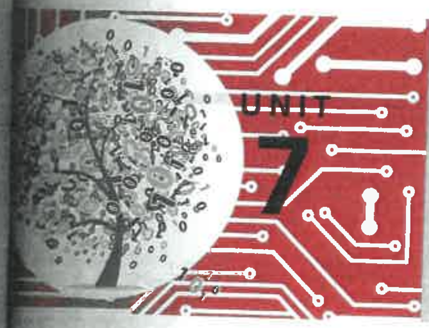
(b) $Z(A, B, C, D, E, F, G) = \Sigma\, m(2, 5, 6, 9) + \Sigma\, d(1, 3, 4, 13, 14)$
$\qquad + E(m_{11} + m_{12}) + F(m_{10}) + G(m_0)$

6.25 (a) Rework Problem 6.6(a), using a five-variable map.

(b) Rework Problem 6.6(a), using the Quine-McCluskey method. Note that you must express F in terms of minterms of all five variables; the original four-variable minterms cannot be used.

6.26 Using map-entered variables, find the minimum sum-of-products expressions for the following function:

$$G = C'E'F + DEF + AD'E'F' + BDE'F + AD'EF'$$



Multi-L
NAND a

Objectives

1. Design a minimal tv ize a given function circuits with an AN
2. Design or analyze a (AND-OR, NAND AND-NOR, and N
3. Design or analyze a
4. Convert circuits of gates, and conversel
5. Design a minimal NAND, or NOR-NC