Formulas

$v_{\text{ave}} = \dfrac{\Delta x}{\Delta t}$	v_{ave} = average velocity Δx = displacement Δt = elapsed time	The definition of average velocity.
$v_{\text{ave}} = \dfrac{(v_{\text{i}} + v_{\text{f}})}{2}$	v_{ave} = average velocity v_{i} = initial velocity v_{f} = final velocity	Another definition of the average velocity, which works when a is constant.
$a = \dfrac{\Delta v}{\Delta t}$	a = acceleration Δv = change in velocity Δt = elapsed time	The definition of acceleration.
$\Delta x = v_{\text{i}}\Delta t + \frac{1}{2}a(\Delta t)^2$	Δx = displacement v_{i} = initial velocity Δt = elapsed time a = acceleration	Use this formula when you don't have v_{f}.
$\Delta x = v_{\text{f}}\Delta t - \frac{1}{2}a(\Delta t)^2$	Δx = displacement v_{f} = final velocity Δt = elapsed time a = acceleration	Use this formula when you don't have v_{i}.
$v_{\text{f}}^2 = v_{\text{i}}^2 + 2a\Delta x$	v_{f} = final velocity v_{i} = initial velocity a = acceleration Δx = displacement	Use this formula when you don't have Δt.
$F = ma$	F = force m = mass a = acceleration	Newton's Second Law. Here, F is the *net* force on the mass m.
$W = mg$	W = weight m = mass g = acceleration due to gravity	The weight of an object with mass m. This is really just Newton's Second Law again.
$f = \mu N$	f = friction force μ = coefficient of friction N = normal force	The "Physics is Fun" equation. Here, μ can be either the kinetic coefficient of friction μ_k or the static coefficient of friction μ_s.
$W = Fd\cos\theta$ *or* $W = F_{\parallel}d$	W = work F = force d = distance θ = angle between F and the direction of motion $F_{\parallel}$ = parallel force	Work is done when a force is applied to an object as it moves a distance d. $F_{\parallel}$ is the component of F in the direction that the object is moved.
$\text{KE} = \frac{1}{2}mv^2$	KE = kinetic energy m = mass v = velocity	The definition of kinetic energy for a mass m with velocity v.
$\text{E} = \text{KE} + \text{PE}$	E = total energy KE = kinetic energy PE = potential energy	The definition of total ("mechanical") energy. If there is no friction, it is conserved (stays constant).
$W = \Delta(\text{KE})$	W = work done KE = kinetic energy	The "work-energy" theorem: the work done by the *net* force on an object equals the change in kinetic energy of the object.
$a_{\text{c}} = \dfrac{v^2}{r}$	a_{c} = centripetal acceleration v = velocity r = radius	The "centripetal" acceleration for an object moving around in a circle of radius r at velocity v.

Vector components

$v_x = v\cos\theta$, $v = \sqrt{v_x^2 + v_y^2}$ $v_y = v\sin\theta$, $\tan\theta = \dfrac{v_y}{v_x}$	For a vector of magnitude v making an angle θ with the x-axis

Inclined Planes

$F_{incline} = mg\sin\theta$ $F_{normal} = mg\cos\theta$	θ is the angle between the inclined plane and the horizontal surface

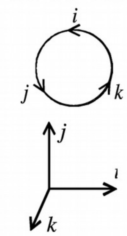
Multiplication of Vectors:

Cross Product or Vector Product:

$i \times j = k$ $\quad j \times i = -k$

$i \times i = 0$

Positive direction:



Dot Product or Scalar Product:

$i \cdot j = 0$ $\quad i \cdot i = 1$

$\mathbf{a} \cdot \mathbf{b} = ab\cos\theta$

Gravitational potential energy $U = mgh$ (local) $U = -\dfrac{GMm}{r}$ (general)	Potential energy U is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level.
Work-energy Theorem $W_{nc} = \Delta K + \Delta U + \Delta E_i$	The work due to non-conservative forces W_{nc} is equal to the change in kinetic energy ΔK plus the change in gravitational potential energy ΔU plus any changes in internal energy due to friction.
Conservation of Mechanical Energy (Only holds true if non-conservative forces are ignored) $E_2 = E_1$ $K_2 + U_2 = K_1 + U_1$	The total mechanical energy of a system, remains constant as the object moves, provided that the net work done by external non-conservative forces (such as friction and air resistance) is zero.

Electric Potential due to a Point Charge: [volts V]

$V = k\dfrac{q}{r}$

V = potential [volts V]
$k = 8.99 \times 10^9$ [N·m²/C²]
q = charge [C]
r = distance [m]

Charge per unit Area: [C/m²]

$\sigma = \dfrac{q}{A}$

σ = charge per unit area [C/m²]
q = charge [C]
A = area [m²]

Electric field due to a point charge q at a distance r $E = k\dfrac{Q}{r^2}$	E is a vector and points away from a positive charge and toward a negative charge.
Electric potential energy $U = k\dfrac{Q_1Q_2}{r}$	The potential energy stored between the interaction between two point charges.
Electric potential $V = k\dfrac{Q}{r}$	The electric potential V due to a point charge q at a distance r away from the charge.
In constant electric fields $\vec{F} = q\vec{E}$ $\quad U = qEd$ $V = Ed$ $\quad U = Vq$	Note that the force F is in the same direction as the electric field E if the charge q is positive and in the opposite direction if the charge is negative. The energy gained by some charge in a field is simply force times the distance traveled. Potential is the energy per unit charge.
Force on a charge moving in a magnetic field $\vec{F} = q\vec{v} \times \vec{B}$ $F = qvB\sin\theta$	A charge q moving in a magnetic field $\vec{B}$ with a velocity $\vec{v}$ experiences a force $\vec{F}$. The magnitude of this force can also be expressed in terms of the angle θ between $\vec{v}$ and $\vec{B}$.

Capacitors in series C_S and parallel C_P $\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ $C_P = C_1 + C_2$	For more than two capacitors: $\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \dfrac{1}{C_4} + \ldots$ $C_P = C_1 + C_2 + C_3 + C_4 + \ldots$
Electric energy stored by a capacitor $U_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\dfrac{Q^2}{C}$	Amount of electric energy stored in a capacitor is given in terms of the capacitance C and the potential difference between the conductors V.

$F_c = \frac{mv^2}{r}$	F_c = centripetal force m = mass v = velocity r = radius	The "centripetal" force that is needed to keep an object of mass m moving around in a circle of radius r at velocity v.
$v = \frac{2\pi r}{T}$	v = velocity r = radius T = period	This formula gives the velocity v of an object moving once around a circle of radius r in time T (the period).
$f = \frac{1}{T}$	f = frequency T = period	The frequency is the number of times per second that an object moves around a circle.
$\tau = rF\sin\theta$ *or* $\tau = rF_\perp$	τ = torque r = distance (radius) F = force θ = angle between F and the lever arm $F_\perp$ = perpendicular force	Torque is a force applied at a distance r from the axis of rotation. $F_\perp = F\sin\theta$ is the component of F perpendicular to the lever arm.
$L = mvr$	L = angular momentum m = mass v = velocity r = radius	Angular momentum is conserved (i.e., it stays constant) as long as there are no external torques.
$F_s = kx$	F_s = spring force k = spring constant x = spring stretch or compression	"Hooke's Law". The force is opposite to the stretch or compression direction.
$PE_s = \frac{1}{2}kx^2$	PE_s = potential energy k = spring constant x = amount of spring stretch or compression	The potential energy stored in a spring when it is either stretched or compressed. Here, $x = 0$ corresponds to the "natural length" of the spring.
$F_g = G\frac{m_1 m_2}{r^2}$	F_g = force of gravity G = a constant m_1, m_2 = masses r = distance of separation	Newton's Law of Gravitation: this formula gives the attractive force between two masses a distance r apart.
$F_e = k\frac{q_1 q_2}{r^2}$	F_e = electric force k = a constant q_1, q_2 = charges r = distance of separation	"Coulomb's Law". This formula gives the force of attraction or repulsion between two charges a distance r apart.
$F = qE$	F = electric force E = electric field q = charge	A charge q, when placed in an electric field E, will feel a force on it, given by this formula (q is sometimes called a "test" charge, since it tests the electric field strength).
$E = \frac{V}{d}$	E = electric field V = voltage d = distance	Between two large plates of metal separated by a distance d which are connected to a battery of voltage V, a uniform electric field between the plates is set up, as given by this formula.
$\Delta V = \frac{W}{q}$	ΔV = potential difference W = work q = charge	The potential difference ΔV between two points (say, the terminals of a battery), is defined as the work per unit charge needed to move charge q from one point to the other.
$V = IR$	V = voltage I = current R = resistance	"Ohm's Law". This law gives the relationship between the battery voltage V, the current I, and the resistance R in a circuit.

Resistivity: [Ohm Meters]

$$\rho = \frac{E}{J}$$

$$\rho = \frac{RA}{L}$$

ρ = resistivity [$\Omega \cdot m$]
E = electric field [N/C]
J = current density [A/m^2]
R = resistance [Ω ohms]
A = area [m^2]
L = length of conductor [m]

Variation of Resistance with Temperature:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

ρ = resistivity [$\Omega \cdot m$]
ρ_0 = reference resistivity [$\Omega \cdot m$]
α = temperature coefficient of resistivity [K^{-1}]
T_0 = reference temperature
$T - T_0$ = temperature difference [K or $°C$]

Charged Particle in a Magnetic Field:

$$r = \frac{mv}{qB}$$

r = radius of rotational path
m = mass [kg]
v = velocity [m/s]
q = charge [C]
B = magnetic field [T]

Magnetic Field Around a Wire: [T]

$$B = \frac{\mu_0 I}{2\pi r}$$

B = magnetic field [T]
μ_0 = the permeability of free space $4\pi \times 10^{-7}$ T·m/A
I = current [A]
r = distance from the center of the conductor

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Avogadro's constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann's constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan–Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Coulomb constant	k	$8.99 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$
Speed of light in vacuum	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass	m_e	$9.110 \times 10^{-31}\,\mathrm{kg} = 0.000549\,\mathrm{u} = 0.511\,\mathrm{MeV\,c^{-2}}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 1.007276\,\mathrm{u} = 938\,\mathrm{MeV\,c^{-2}}$
Neutron rest mass	m_n	$1.675 \times 10^{-27}\,\mathrm{kg} = 1.008665\,\mathrm{u} = 940\,\mathrm{MeV\,c^{-2}}$
Unified atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$

Metric (SI) multipliers

Prefix	Abbreviation	Value
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

$P = IV$ *or* $P = V^2/R$ *or* $P = I^2R$	P = power I = current V = voltage R = resistance	All of these power formulas are equivalent and give the power used in a circuit resistor R. Use the formula that has the quantities that you know.
$R_s = R_1 + R_2 + \ldots$	R_s = total (series) resistance R_1 = first resistor R_2 = second resistor ...	When resistors are placed end to end, which is called "in series", the effective total resistance is just the sum of the individual resistances.
$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$	R_p = total (parallel) resistance R_1 = first resistor R_2 = second resistor ...	When resistors are placed side by side (or "in parallel"), the effective total resistance is the inverse of the sum of the reciprocals of the individual resistances (whew!).
$q = CV$	q = charge C = capacitance V = voltage	This formula is "Ohm's Law" for capacitors. Here, C is a number specific to the capacitor (like R for resistors), q is the charge on one side of the capacitor, and V is the voltage across the capacitor.
$F = ILB \sin\theta$	F = force on a wire I = current in the wire L = length of wire B = external magnetic field θ = angle between the current direction and the magnetic field	This formula gives the force on a wire carrying current I while immersed in a magnetic field B. Here, θ is the angle between the direction of the current and the direction of the magnetic field (θ is usually 90°, so that the force is $F = ILB$).
$v = \lambda f$	v = wave velocity λ = wavelength f = frequency	This formula relates the wavelength and the frequency of a wave to its speed. The formula works for both sound and light waves.
$v = \frac{c}{n}$	v = velocity of light c = vacuum light speed n = index of refraction	When light travels through a medium (say, glass), it slows down. This formula gives the speed of light in a medium that has an index of refraction n. Here, $c = 3.0 \times 10^8$ m/s.
$n_1 \sin\theta_1 = n_2 \sin\theta_2$	n_1 = incident index θ_1 = incident angle n_2 = refracted index θ_2 = refracted angle	"Snell's Law". When light moves from one medium (say, air) to another (say, glass) with a different index of refraction n, it changes direction (refracts). The angles are taken from the normal (per-
$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$	d_o = object distance d_i = image distance f = focal length	This formula works for lenses and mirrors, and relates the focal length, object distance, and image distance.
$m = -\frac{d_i}{d_o}$	m = magnification d_i = image distance d_o = object distance	The magnification m is how much bigger ($\|m\| > 1$) or smaller ($\|m\| < 1$) the image is compared to the object. If $m < 0$, the image is inverted compared to the object.

Total internal reflection

$$\sin\theta_c = \frac{n_2}{n_1}$$

The critical angle θ_c is the angle of incidence beyond which total internal reflection occurs. The index of refraction for the medium in which the incident ray is traveling is n_1

$$density = \frac{mass}{volume} \qquad \rho = \frac{m}{V}$$

pressure × volume = number of moles × molar gas constant × absolute temperature $\qquad pV = nRT$

$$\frac{1}{\text{Focal length}} = \frac{1}{\text{distance to image}} + \frac{1}{\text{distance to object}} \qquad \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

Velocity = (frequency)(wavelength) $\qquad v = f\lambda$

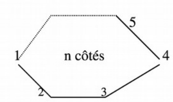
$$Y = mx + P$$
$$m = \frac{Y_2 - Y_1}{x_2 - x_1} \qquad x = \frac{Y - P}{m}$$
$$P = Y_1 - mx_1$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t = \gamma \Delta t_0$$
$$L = \frac{L_0}{\gamma}$$

$$m_1x + P_1 = m_2x + P_2$$
$$m_1x = m_2x + P_2 - P_1$$
$$m_1x - m_2x = P_2 - P_1$$
$$x(m_1 - m_2) = P_2 - P_1$$
$$x = \frac{P_2 - P_1}{m_1 - m_2}$$

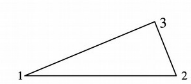
$$\frac{Y - P_1}{m_1} = \frac{Y - P_2}{m_2}$$
$$Ym_2 - P_1m_2 = Ym_1 - P_2m_1$$
$$Ym_2 - Ym_1 = P_1m_2 - P_2m_1$$
$$Y = \frac{P_1m_2 - P_2m_1}{m_2 - m_1}$$

→Les angles d'un Polygone



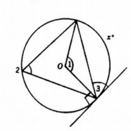
∠1+∠2+∠3+∠4+∠5………=(n-2)180°

Exemple : *Le cas d'un triangle*



∠1+∠2+∠3=180°

→Les angles dans un cercle

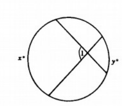


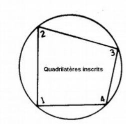
∠1= x°

∠2= $\frac{1}{2}$ x°

∠3 = x°

∠1= $\frac{x° + y°}{2}$

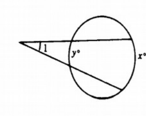




∠1+∠3=180°

∠2+∠4=180°

∠1= $\frac{x° - y°}{2}$



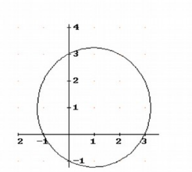
→L'équation d'un cercle de centre (x₁,y₁) et de rayon r est :

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

Exemple :

L'équation du cercle de centre (1,2) et de rayon 3 est :

$$(x-1)^2 + (y-2)^2 = 9$$



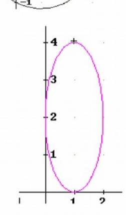
→L'équation d'une ellipse de centre (x₁,y₁) et d'axes a et b est :

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} = 1$$

Exemple:

L'équation de l'ellipse de centre (3,2) et d'axes 1 et 2 est :

$$\frac{(x-3)^2}{1} + \frac{(y-2)^2}{4} = 1$$



$Q = mc\,\Delta T$	Q = heat added or removed m = mass of substance c = specific heat ΔT = change in temperature	The specific heat c for a substance gives the heat needed to raise the temperature of a mass m of that substance by ΔT degrees. If $\Delta T < 0$, the formula gives the heat that has to be *removed* to lower the temperature.
$Q = ml$	Q = heat added or removed m = mass of substance l = specific heat of transformation	When a substance undergoes a change of phase (for example, when ice melts), the temperature doesn't change; however, heat has to be added (ice melting) or removed (water freezing). The specific heat of transformation l is different for each substance.
$\Delta U = Q - W$	ΔU = change in internal energy Q = heat added W = work done by the system	The "first law of thermodynamics". The change in internal energy of a system is the heat added minus the work done by the system.
$E_{\text{eng}} = \dfrac{W}{Q_{\text{hot}}} \times 100$	E_{eng} = % efficiency of the heat engine W = work done by the engine Q_{hot} = heat absorbed by the engine	A heat engine essentially converts heat into work. The engine does work by absorbing heat from a hot reservoir and discarding some heat to a cold reservoir. The formula gives the quality ("efficiency") of the engine.
$P = \dfrac{F}{A}$	P = pressure F = force A = area	The definition of pressure. P is a force per unit area exerted by a gas or fluid on the walls of the container.
$\dfrac{PV}{T} = \text{constant}$	P = pressure V = volume T = temperature	The "Ideal Gas Law". For "ideal" gases (and also for real-life gases at low pressure), the pressure of the gas times the volume of the gas divided by the temperature of the gas is a constant.
$E = hf$	E = photon energy h = a constant f = wave frequency	The energy of a photon is proportional to its wave frequency; h is a number called "Planck's constant".
$\lambda = \dfrac{h}{p}$	λ = matter wavelength h = a constant p = momentum	A particle can act like a wave with wavelength λ, as given by this formula, if it has momentum p. This is called "wave-particle" duality.
$\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$	γ = the relativistic factor v = speed of moving observer c = speed of light	The relativistic factor γ is the amount by which moving clocks slow down and lengths contract, as seen by an observer compared to those of another observer moving at speed v (note that $\gamma \geq 1$).

Shape	Formulas for Area (A) and Circumference (C)
Triangle	$A = \frac{1}{2}bh = \frac{1}{2} \times$ base $\times$ height
Rectangle	$A = lw$ = length $\times$ width
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2} \times$ sum of bases $\times$ height
Parallelogram	$A = bh$ = base $\times$ height
Circle	$A = \pi r^2 = \pi \times$ square of radius $C = 2\pi r = 2 \times \pi \times$ radius
Figure	**Formulas for Volume (V) and Surface Area (SA)**
Rectangular Prism	$V = lwh$ = length $\times$ width $\times$ height $SA = 2lw + 2hw + 2lh$ = 2(length $\times$ width) + 2(height $\times$ width) + 2(length $\times$ height)
General Prisms	$V = Bh$ = area of base $\times$ height SA = sum of the areas of the faces
Right Circular Cylinder	$V = Bh$ = area of base $\times$ height $SA = 2B + Ch$ = (2 $\times$ area of base) + (circumference $\times$ height)
Right Pyramid	$V = \frac{1}{3}Bh = \frac{1}{3} \times$ area of base $\times$ height $SA = B + \frac{1}{2}P\ell$ = area of base + ($\frac{1}{2}$ $\times$ perimeter of base $\times$ slant height)
Right Circular Cone	$V = \frac{1}{3}Bh = \frac{1}{3} \times$ area of base $\times$ height $SA = B + \frac{1}{2}C\ell$ = area of base + ($\frac{1}{2}$ $\times$ circumference $\times$ slant height)
Sphere	$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times$ cube of radius $SA = 4\pi r^2 = 4 \times \pi \times$ square of radius

Coordinate Geometry Formulas

Let (x_1, y_1) and (x_2, y_2) be two points in the plane.

slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$

midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Interest Formulas

Simple Interest: $A = P(1 + rt)$

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = amount (including interest)
P = principal
r = interest rate (expressed as a decimal)
n = number of compoundings per year
t = number of years

Quadratic Equations

Let $ax^2 + bx + c = 0$, where $a \neq 0$.

Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x-coordinate of vertex $= -\dfrac{b}{2a}$

$$a^{\log_a M} = M$$

$$\log_a a^m = m$$

$$\log_a M = \frac{\ln M}{\ln a}$$

$$\log_a(M \cdot N) = \log_a(M) + \log_a(N)$$

$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_b(N)$$

$$\log_a M^r = r \cdot \log_a M$$

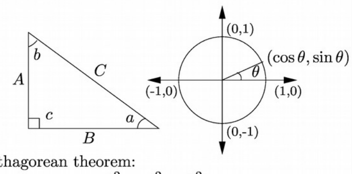
$$\log_a(a) = 1$$

$$\det(\boldsymbol{A} \cdot \boldsymbol{B}) = \det(\boldsymbol{A})\det(\boldsymbol{B})$$

$$\det\begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{A} \end{pmatrix} = \det(\boldsymbol{A} + \boldsymbol{B})\det(\boldsymbol{A} - \boldsymbol{B})$$

Theoretical Computer Science Cheat Sheet

Trigonometry



Pythagorean theorem:
$$C^2 = A^2 + B^2.$$

Definitions:
$$\sin a = A/C, \quad \cos a = B/C,$$
$$\csc a = C/A, \quad \sec a = C/B,$$
$$\tan a = \frac{\sin a}{\cos a} = \frac{A}{B}, \quad \cot a = \frac{\cos a}{\sin a} = \frac{B}{A}.$$

Area, radius of inscribed circle:
$$\tfrac{1}{2}AB, \quad \frac{AB}{A+B+C}.$$

Identities:
$$\sin x = \frac{1}{\csc x}, \qquad \cos x = \frac{1}{\sec x},$$
$$\tan x = \frac{1}{\cot x}, \qquad \sin^2 x + \cos^2 x = 1,$$
$$1 + \tan^2 x = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x,$$
$$\sin x = \cos\left(\tfrac{\pi}{2} - x\right), \qquad \sin x = \sin(\pi - x),$$
$$\cos x = -\cos(\pi - x), \qquad \tan x = \cot\left(\tfrac{\pi}{2} - x\right),$$
$$\cot x = -\cot(\pi - x), \qquad \csc x = \cot\tfrac{x}{2} - \cot x,$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y,$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y,$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$$
$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot x \pm \cot y},$$
$$\sin 2x = 2 \sin x \cos x, \qquad \sin 2x = \frac{2 \tan x}{1 + \tan^2 x},$$
$$\cos 2x = \cos^2 x - \sin^2 x, \qquad \cos 2x = 2\cos^2 x - 1,$$
$$\cos 2x = 1 - 2\sin^2 x, \qquad \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x},$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}, \qquad \cot 2x = \frac{\cot^2 x - 1}{2 \cot x},$$
$$\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y,$$
$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y.$$

Euler's equation:
$$e^{ix} = \cos x + i \sin x, \qquad e^{i\pi} = -1.$$

Matrices

Multiplication:
$$C = A \cdot B, \quad c_{i,j} = \sum_{k=1}^{n} a_{i,k} b_{k,j}.$$

Determinants: $\det A \neq 0$ iff A is non-singular.
$$\det A \cdot B = \det A \cdot \det B,$$
$$\det A = \sum_{\pi} \prod_{i=1}^{n} \operatorname{sign}(\pi) a_{i,\pi(i)}.$$

2×2 and 3×3 determinant:
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = g \begin{vmatrix} b & c \\ e & f \end{vmatrix} - h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + i \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$
$$= \frac{aei + bfg + cdh}{- ceg - fha - ibd.}$$

Permanents:
$$\operatorname{perm} A = \sum_{\pi} \prod_{i=1}^{n} a_{i,\pi(i)}.$$

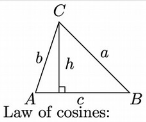
Hyperbolic Functions

Definitions:
$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2},$$
$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \qquad \operatorname{csch} x = \frac{1}{\sinh x},$$
$$\operatorname{sech} x = \frac{1}{\cosh x}, \qquad \coth x = \frac{1}{\tanh x}.$$

Identities:
$$\cosh^2 x - \sinh^2 x = 1, \qquad \tanh^2 x + \operatorname{sech}^2 x = 1,$$
$$\coth^2 x - \operatorname{csch}^2 x = 1, \qquad \sinh(-x) = -\sinh x,$$
$$\cosh(-x) = \cosh x, \qquad \tanh(-x) = -\tanh x,$$
$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y,$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y,$$
$$\sinh 2x = 2 \sinh x \cosh x,$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x,$$
$$\cosh x + \sinh x = e^x, \qquad \cosh x - \sinh x = e^{-x},$$
$$(\cosh x + \sinh x)^n = \cosh nx + \sinh nx, \quad n \in \mathbb{Z},$$
$$2\sinh^2 \tfrac{x}{2} = \cosh x - 1, \qquad 2\cosh^2 \tfrac{x}{2} = \cosh x + 1.$$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

More Trig.



Law of cosines:
$$c^2 = a^2 + b^2 - 2ab \cos C.$$

Area:
$$A = \tfrac{1}{2}hc,$$
$$= \tfrac{1}{2}ab \sin C,$$
$$= \frac{c^2 \sin A \sin B}{2 \sin C}.$$

Heron's formula:
$$A = \sqrt{s \cdot s_a \cdot s_b \cdot s_c},$$
$$s = \tfrac{1}{2}(a + b + c),$$
$$s_a = s - a,$$
$$s_b = s - b,$$
$$s_c = s - c.$$

More identities:
$$\sin\tfrac{x}{2} = \sqrt{\frac{1 - \cos x}{2}},$$
$$\cos\tfrac{x}{2} = \sqrt{\frac{1 + \cos x}{2}},$$
$$\tan\tfrac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}},$$
$$= \frac{1 - \cos x}{\sin x},$$
$$= \frac{\sin x}{1 + \cos x},$$
$$\cot\tfrac{x}{2} = \sqrt{\frac{1 + \cos x}{1 - \cos x}},$$
$$= \frac{1 + \cos x}{\sin x},$$
$$= \frac{\sin x}{1 - \cos x},$$
$$\sin x = \frac{e^{ix} - e^{-ix}}{2i},$$
$$\cos x = \frac{e^{ix} + e^{-ix}}{2},$$
$$\tan x = -i \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}},$$
$$= -i \frac{e^{2ix} - 1}{e^{2ix} + 1},$$
$$\sin x = \frac{\sinh ix}{i},$$
$$\cos x = \cosh ix,$$
$$\tan x = \frac{\tanh ix}{i}.$$

Graph Theory

Definitions:

Loop	An edge connecting a vertex to itself.
Directed	Each edge has a direction.
Simple	Graph with no loops or multi-edges.
Walk	A sequence $v_0 e_1 v_1 \ldots e_\ell v_\ell$.
Trail	A walk with distinct edges.
Path	A trail with distinct vertices.
Connected	A graph where there exists a path between any two vertices.
Component	A maximal connected subgraph.
Tree	A connected acyclic graph.
Free tree	A tree with no root.
DAG	Directed acyclic graph.
Eulerian	Graph with a trail visiting each edge exactly once.
Hamiltonian	Graph with a cycle visiting each vertex exactly once.
Cut	A set of edges whose removal increases the number of components.
Cut-set	A minimal cut.
Cut edge	A size 1 cut.
k-Connected	A graph connected with the removal of any $k - 1$ vertices.
k-Tough	$\forall S \subseteq V, S \neq \emptyset$ we have $k \cdot c(G - S) \leq \lvert S \rvert$.
k-Regular	A graph where all vertices have degree k.
k-Factor	A k-regular spanning subgraph.
Matching	A set of edges, no two of which are adjacent.
Clique	A set of vertices, all of which are adjacent.
Ind. set	A set of vertices, none of which are adjacent.
Vertex cover	A set of vertices which cover all edges.
Planar graph	A graph which can be embeded in the plane.
Plane graph	An embedding of a planar graph.

$$\sum_{v \in V} \deg(v) = 2m.$$

If G is planar then $n - m + f = 2$, so
$$f \leq 2n - 4, \quad m \leq 3n - 6.$$

Any planar graph has a vertex with degree ≤ 5.

Notation:

$E(G)$	Edge set
$V(G)$	Vertex set
$c(G)$	Number of components
$G[S]$	Induced subgraph
$\deg(v)$	Degree of v
$\Delta(G)$	Maximum degree
$\delta(G)$	Minimum degree
$\chi(G)$	Chromatic number
$\chi_E(G)$	Edge chromatic number
G^c	Complement graph
K_n	Complete graph
K_{n_1,n_2}	Complete bipartite graph
$\mathrm{r}(k,\ell)$	Ramsey number

Geometry

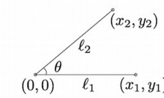
Projective coordinates: triples (x, y, z), not all x, y and z zero.
$$(x, y, z) = (cx, cy, cz) \quad \forall c \neq 0.$$

Cartesian	Projective
(x, y)	$(x, y, 1)$
$y = mx + b$	$(m, -1, b)$
$x = c$	$(1, 0, -c)$

Distance formula, L_p and L_∞ metric:
$$\sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2},$$
$$\left[|x_1 - x_0|^p + |y_1 - y_0|^p\right]^{1/p},$$
$$\lim_{p \to \infty} \left[|x_1 - x_0|^p + |y_1 - y_0|^p\right]^{1/p}.$$

Area of triangle (x_0, y_0), (x_1, y_1) and (x_2, y_2):
$$\tfrac{1}{2} \operatorname{abs} \begin{vmatrix} x_1 - x_0 & y_1 - y_0 \\ x_2 - x_0 & y_2 - y_0 \end{vmatrix}.$$

Angle formed by three points:



$$\cos\theta = \frac{(x_1, y_1) \cdot (x_2, y_2)}{\ell_1 \ell_2}.$$

Line through two points (x_0, y_0) and (x_1, y_1):
$$\begin{vmatrix} x & y & 1 \\ x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0.$$

X	0rad.=0°	$\frac{\pi}{6}$ rad.=30°	$\frac{\pi}{4}$ rad.=45°	$\frac{\pi}{3}$ rad.=60°	$\frac{\pi}{2}$ rad.=90°
sin(x)	$\frac{\sqrt{0}}{2}=\mathbf{0}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=\mathbf{1}$
cos(x)	$\frac{\sqrt{4}}{2}=\mathbf{1}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{0}}{2}=\mathbf{0}$

$$\sin(x)\sin(y) = \tfrac{1}{2}\left(-\cos(x+y) + \cos(x-y)\right); \qquad \sin(x)\cos(y) = \tfrac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$$
$$\cos(x)\sin(y) = \tfrac{1}{2}\left(\sin(x+y) - \sin(x-y)\right); \qquad \cos(x)\cos(y) = \tfrac{1}{2}\left(\cos(x+y) + \cos(x-y)\right)$$

π

Wallis' identity:
$$\pi = 2 \cdot \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdots}$$

Brouncker's continued fraction expansion:
$$\frac{\pi}{4} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cdots}}}}$$

Gregrory's series:
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

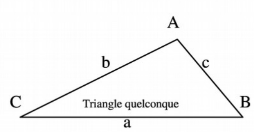
Newton's series:
$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2 \cdot 3 \cdot 2^3} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 2^5} + \cdots$$

Sharp's series:
$$\frac{\pi}{6} = \frac{1}{\sqrt{3}}\left(1 - \frac{1}{3^1 \cdot 3} + \frac{1}{3^2 \cdot 5} - \frac{1}{3^3 \cdot 7} + \cdots\right)$$

Euler's series:
$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots$$
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \cdots$$
$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \cdots$$

<u>Loi des Cosinus</u>

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$



<u>Loi des Sinus</u>

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

• *Sum and Difference of Angles*

$$\operatorname{sh}(x \pm y) = \operatorname{sh}(x)\operatorname{ch}(y) \pm \operatorname{ch}(x)\operatorname{sh}(y); \qquad \operatorname{ch}(x \pm y) = \operatorname{ch}(x)\operatorname{ch}(y) \pm \operatorname{sh}(x)\operatorname{sh}(y)$$
$$\operatorname{th}(x \pm y) = \frac{\operatorname{th}(x) \pm \operatorname{th}(y)}{1 \pm \operatorname{th}(x)\operatorname{th}(y)}$$

• *Multiples of an Angle*

$$\operatorname{sh}(2x) = 2\operatorname{sh}(x)\operatorname{ch}(x); \qquad \operatorname{sh}\left(\tfrac{x}{2}\right)^2 = \frac{\operatorname{ch}(x) - 1}{2}$$
$$\operatorname{ch}(2x) = \operatorname{sh}(x)^2 + \operatorname{ch}(x)^2; \qquad \operatorname{ch}\left(\tfrac{x}{2}\right)^2 = \frac{\operatorname{ch}(x) + 1}{2}$$
$$\operatorname{th}(2x) = \frac{2\operatorname{th}(x)}{1 + \operatorname{th}(x)^2}; \qquad \operatorname{th}\left(\tfrac{x}{2}\right)^2 = \frac{\operatorname{ch}(x) - 1}{\operatorname{sh}(x)} = \frac{\operatorname{sh}(x)}{\operatorname{ch}(x) + 1}$$

• *Other Identities*

$$\operatorname{ch}(x) = \tfrac{1}{2}\left(e^x + e^{-x}\right); \qquad \operatorname{sh}(x) = \tfrac{1}{2}\left(e^x - e^{-x}\right)$$
$$\operatorname{ch}(x)^2 - \operatorname{sh}(x)^2 = 1; \qquad \operatorname{th}(x)^2 + \frac{1}{\operatorname{ch}(x)^2} = 1$$

2.1.3 Products

$$\prod_{k=1}^{\infty}\left(1 - \left(\tfrac{x}{\pi k}\right)^2\right) = \prod_{k=1}^{\infty} \cos\left(\tfrac{x}{2^k}\right) = \frac{\sin(x)}{x}; \qquad \prod_{k=1}^{\infty}\left(1 - \frac{4x^2}{(2k-1)^2}\right) = \cos(\pi x)$$
$$\prod_{k=0}^{\infty}\left(1 + x^{2^k}\right) = \frac{1}{1-x}; \qquad \prod_{k=1}^{\infty}\left(1 + \frac{(-1)^{k+1}}{2k-1}\right) = \sqrt{2}$$

2.1.4 Sums

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{6}\pi^2; \qquad \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{1}{90}\pi^4$$
$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{1}{8}\pi^2; \qquad \sum_{k=1}^{\infty} \frac{1}{(2k-1)^4} = \frac{1}{96}\pi^4$$
$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{1}{4}\pi; \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln(2)$$

$$\sum_{k=1}^{n} k = \tfrac{1}{2}n(n+1); \qquad \sum_{k=1}^{n} k^2 = \tfrac{1}{6}n(n+1)(2n+1)$$
$$\sum_{k=1}^{n} k^3 = \left(\tfrac{1}{2}n(n+1)\right)^2; \qquad \sum_{k=1}^{n}\left(\frac{1}{k^p} - \frac{1}{(k+1)^p}\right) = 1 - \frac{1}{(n+1)^p}, \quad p \in \mathbb{R}$$
$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x}; \qquad \sum_{k=0}^{n} \binom{m+k}{m} = \binom{m+n+1}{m+1}$$
$$\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n$$

Series

Taylor's series:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \cdots = \sum_{i=0}^{\infty} \frac{(x-a)^i}{i!} f^{(i)}(a).$$

Expansions:

$$
\begin{array}{lll}
\frac{1}{1-x} & = 1 + x + x^2 + x^3 + x^4 + \cdots & = \sum_{i=0}^{\infty} x^i, \\
\frac{1}{1-cx} & = 1 + cx + c^2x^2 + c^3x^3 + \cdots & = \sum_{i=0}^{\infty} c^i x^i, \\
\frac{1}{1-x^n} & = 1 + x^n + x^{2n} + x^{3n} + \cdots & = \sum_{i=0}^{\infty} x^{ni}, \\
\frac{x}{(1-x)^2} & = x + 2x^2 + 3x^3 + 4x^4 + \cdots & = \sum_{i=0}^{\infty} i x^i, \\
x^k \frac{d^n}{dx^n}\left(\frac{1}{1-x}\right) & = x + 2^n x^2 + 3^n x^3 + 4^n x^4 + \cdots & = \sum_{i=0}^{\infty} i^n x^i, \\
e^x & = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots & = \sum_{i=0}^{\infty} \frac{x^i}{i!}, \\
\ln(1+x) & = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 - \cdots & = \sum_{i=1}^{\infty} (-1)^{i+1}\frac{x^i}{i}, \\
\ln\frac{1}{1-x} & = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots & = \sum_{i=1}^{\infty} \frac{x^i}{i}, \\
\sin x & = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots & = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!}, \\
\cos x & = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots & = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!}, \\
\tan^{-1} x & = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots & = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)}, \\
(1+x)^n & = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots & = \sum_{i=0}^{\infty} \binom{n}{i} x^i, \\
\frac{1}{(1-x)^{n+1}} & = 1 + (n+1)x + \binom{n+2}{2}x^2 + \cdots & = \sum_{i=0}^{\infty} \binom{i+n}{i} x^i, \\
\frac{x}{e^x - 1} & = 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \cdots & = \sum_{i=0}^{\infty} \frac{B_i x^i}{i!}, \\
\frac{1}{2x}(1 - \sqrt{1-4x}) & = 1 + x + 2x^2 + 5x^3 + \cdots & = \sum_{i=0}^{\infty} \frac{1}{i+1}\binom{2i}{i} x^i, \\
\frac{1}{\sqrt{1-4x}} & = 1 + x + 2x^2 + 6x^3 + \cdots & = \sum_{i=0}^{\infty} \binom{2i}{i} x^i, \\
\frac{1}{\sqrt{1-4x}}\left(\frac{1-\sqrt{1-4x}}{2x}\right)^n & = 1 + (2+n)x + \binom{4+n}{2}x^2 + \cdots & = \sum_{i=0}^{\infty} \binom{2i+n}{i} x^i, \\
\frac{1}{1-x}\ln\frac{1}{1-x} & = x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \cdots & = \sum_{i=1}^{\infty} H_i x^i, \\
\frac{1}{2}\left(\ln\frac{1}{1-x}\right)^2 & = \frac{1}{2}x^2 + \frac{3}{4}x^3 + \frac{11}{24}x^4 + \cdots & = \sum_{i=2}^{\infty} \frac{H_{i-1}x^i}{i}, \\
\frac{x}{1-x-x^2} & = x + x^2 + 2x^3 + 3x^4 + \cdots & = \sum_{i=0}^{\infty} F_i x^i, \\
\frac{F_n x}{1-(F_{n-1}+F_{n+1})x - (-1)^n x^2} & = F_n x + F_{2n}x^2 + F_{3n}x^3 + \cdots & = \sum_{i=0}^{\infty} F_{ni} x^i.
\end{array}
$$

$$
\begin{array}{llll}
\frac{1}{1+x} & = \sum_{k=0}^{\infty}(-1)^k x^k & \simeq & 1 - x + x^2 + \ldots + (-1)^n x^n \\
\frac{1}{(1+x)^2} & = \sum_{k=0}^{\infty}(-1)^k (k+1) x^k & \simeq & 1 - 2x + 3x^2 + \ldots + (-1)^n (n+1) x^n \\
\sqrt{1+x} & = 1 + \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(2k-1)!!}{2^k k!}x^k & \simeq & 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots + (-1)^{n+1}\frac{(2n-3)!!}{2^n n!}x^n \\
\frac{1}{\sqrt{1+x}} & = 1 + \sum_{k=1}^{\infty}(-1)^{k}\frac{(2k-1)!!}{2^k k!}x^k & \simeq & 1 - \frac{x}{2} + \frac{3x^2}{8} + \ldots + (-1)^{n}\frac{(2n-1)!!}{2^n n!}x^n \\
(1+x)^\alpha & = 1 + \sum_{k=1}^{\infty}\frac{\prod_{i=0}^{k-1}(\alpha-i)}{k!}x^k & \simeq & 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \ldots + \frac{\prod_{i=0}^{n-1}(\alpha-i)}{n!}x^n \\
\ln\left(\frac{1+x}{1-x}\right) & = 2\sum_{k=0}^{\infty}\frac{x^{2k+1}}{2k+1} & \simeq & 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2n+1}}{2n+1}\right) \\
\int_0^x dy\, e^{-y^2} & = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{k!(2k+1)} & \simeq & x - \frac{x^3}{3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + \ldots + (-1)^n\frac{x^{2n+1}}{n!(2n+1)}
\end{array}
$$

$$(a+b)^n = a^n + C_1^n a^{n-1}b^1 + C_2^n a^{n-2}b^2 + \ldots\ldots\ldots + C_{n-2}^n a^2 b^{n-2} + C_{n-1}^n a^1 b^{n-1} + b^n$$
$$= \sum_{k=0}^{k=n} C_k^n a^k b^{(n-k)}$$

$$(a-b)^n = \sum_{k=0}^{k=n} (-1)^k C_k^n a^k b^{(n-k)}$$

$$\frac{a}{b} \mp \frac{c}{d} = \frac{a\cdot d \mp b\cdot c}{b\cdot d}; \quad (b\cdot d \neq 0).$$

Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

In general:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n}\left((i+1)^{m+1} - i^{m+1} - (m+1)i^m\right)\right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k n^{m+1-k}.$$

Geometric series:

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, \quad \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, \quad |c| < 1,$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1.$$

Harmonic series:

$$H_n = \sum_{i=1}^{n}\frac{1}{i}, \quad \sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2}H_n - \frac{n(n-1)}{4}.$$

$$\sum_{i=1}^{n} H_i = (n+1)H_n - n, \quad \sum_{i=1}^{n}\binom{i}{m}H_i = \binom{n+1}{m+1}\left(H_{n+1} - \frac{1}{m+1}\right).$$

1. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$, **2.** $\sum_{k=0}^{n}\binom{n}{k} = 2^n$, **3.** $\binom{n}{k} = \binom{n}{n-k}$,

4. $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$, **5.** $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$,

6. $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$, **7.** $\sum_{k=0}^{n}\binom{r+k}{k} = \binom{r+n+1}{n}$,

8. $\sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$, **9.** $\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$,

10. $\binom{n}{k} = (-1)^k\binom{k-n-1}{k}$,

$$P_k^n = \frac{n!}{(n-k)!} \quad \text{(permutation)}$$

$$C_k^n = \binom{n}{k} = \frac{n!}{(n-k)!k!} \quad \text{(Combinaison)}$$

$$C_k^n = C_{k-1}^{n-1} + C_k^{n-1}$$

$$C_k^n = C_{n-k}^n$$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Ordinary power series:

$$A(x) = \sum_{i=0}^{\infty} a_i x^i.$$

Exponential power series:

$$A(x) = \sum_{i=0}^{\infty} a_i \frac{x^i}{i!}.$$

Dirichlet power series:

$$A(x) = \sum_{i=1}^{\infty} \frac{a_i}{i^x}.$$

Binomial theorem:

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k.$$

Difference of like powers:

$$x^n - y^n = (x-y)\sum_{k=0}^{n-1} x^{n-1-k}y^k.$$

Distribution name	Distribution	$\langle X\rangle$	$(\Delta X)^2$
Binomial $b(n,p)$	$p(X=x) = \binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$
Poisson $P(\lambda)$	$p(X=x) = \frac{\lambda^x}{x!}e^{-\lambda}$	λ	λ
Geometric $G(p)$	$p(X=x) = p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Pascal $Pa(n,x,p)$	$p(X=x) = \binom{n-1}{x-1}p^x(1-p)^{n-x}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Exponential $\exp(\lambda)$	$p(X=x) = \begin{cases}\lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0\end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma $\Gamma(t,\lambda)$	$p(X=x) = \begin{cases}\frac{\lambda e^{-\lambda x}(\lambda x)^{t-1}}{\Gamma(s)}, & x \geq 0 \\ 0, & x < 0\end{cases}$	$\frac{t}{\lambda}$	$\frac{t}{\lambda^2}$
Normal $N(\boldsymbol{\mu},\sigma)$	$p(X=\mathbf{x}) = \frac{1}{(2\pi\sigma^2)^{N/2}}e^{-\frac{(\mathbf{x}-\boldsymbol{\mu})^2}{2\sigma^2}}$	$\boldsymbol{\mu}$	σ^2

For events A and B:

$$\Pr[A \vee B] = \Pr[A] + \Pr[B] - \Pr[A \wedge B]$$

$$\Pr[A \wedge B] = \Pr[A]\cdot\Pr[B],$$

iff A and B are independent.

$$\Pr[A|B] = \frac{\Pr[A \wedge B]}{\Pr[B]}$$